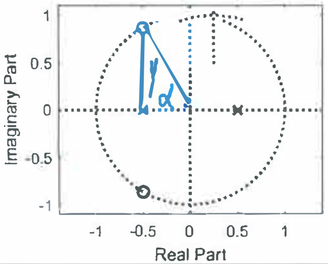
Name *Answers*

EE-3221-41 – Dr. Durant – Quiz 9
Winter 2017-'18, Week 10

z-transform: $X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$ DFT: $X(k) = \sum_{n=0}^{N-1} w_N^{kn} x(n)$, where $w_N = e^{-j\frac{2\pi}{N}}$

1. (2 points) Make a list of zeros and a list of poles given this z-plane view of a system H(z). Note that the real magnitudes are all ½.



$y = \sqrt{1^2 - \left(\frac{1}{2}\right)^2} = \frac{\sqrt{3}}{2}$ $\alpha = \frac{\pi}{3} = 60^0$

$z_k = -\frac{1}{2} \pm j\frac{\sqrt{3}}{2} = 1\angle \pm \frac{\pi}{3}$

$p_k = \pm \frac{1}{2}$

2. (2 points) Given the roots you listed above, write out H(z). Fully expand the numerator and the denominator. Multiply by z^{-1}/z^{-1} as many times as needed to eliminate positive exponents.
3. (2 points) Recall that H(z) = Y(z) / X(z). Take the inverse z-transform of your result in 2 and solve for y(n) to determine the difference equation that implements the system H(z).

(2) $H(z) = \frac{(z - 1\angle\frac{\pi}{3})(z - 1\angle\frac{-\pi}{3})}{(z-\frac{1}{2})(z+\frac{1}{2})} = \frac{z^2 - z(z_1+z_2) + 1}{z^2 - \frac{1}{4}} = \frac{z^2+z+1}{z^2-\frac{1}{4}} \cdot \frac{z^{-2}}{z^{-2}} = \frac{1+z^{-1}+z^{-2}}{1-\frac{1}{4}z^{-2}}$

(3) $H(z) = \frac{Y(z)}{X(z)} = \frac{1-z^{-1}+z^{-2}}{1-\frac{1}{4}z^{-2}}$

$Y(z) - \frac{1}{4}z^{-2}Y(z) = X(z) + z^{-1}X(z) + z^{-2}X(z)$

$y(n) - \frac{1}{4}y(n-2) = x(n) + x(n-1) + x(n-2)$

$y(n) = \frac{1}{4}y(n-2) + x(n) + x(n-1) + x(n-2)$

4. (2 points) In MATLAB, x = [5 6 1 6] and h = [4 1 2 4]. y = conv(h,x) is executed and correctly gives y = [20 29 20 57 32 16 24]. We attempt to perform the convolution in the DFT domain, y2 = ifft(fft(h).*fft(x)). This not only gives the wrong answer, but it gives an answer of the wrong length. Explain what happened and calculate the result returned in y2.

5. (2 points) Calculate the DFT value X(2) for x = [1 2 3 4].

(4) Circular convolution of period $N=4$. Aliasing of $x(4)$ over $x(0)$, $x(5)$ over $x(1)$, $x(6)$ over $x(2)$.

$y2 =$ ~~20 29 57 32~~

$$\begin{array}{rrrrl} & 20 & 29 & 20 & 57 \quad \leftarrow N=0..3 \\ + & 32 & 16 & 24 & \quad\quad N=4..7 \quad \therefore \quad N\%4=0..3 \\ \hline \rightarrow [& 52 & 45 & 44 & 57] \end{array}$$

(5) $w_4 = e^{-j\frac{2\pi}{4}} = -j$

$k=2$, so step is $w_4^2 = -1$

$\therefore$ signal to correlate is $[1 \ -1 \ 1 \ -1]$

Take dot product: $[1 \ 2 \ 3 \ 4] \cdot [1 \ -1 \ 1 \ -1] = 1-2+3-4 = \boxed{-2}$

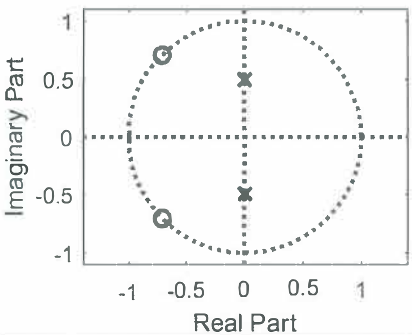
Name Answers

EE-3221-11 – Dr. Durant – Quiz 9
Winter 2017-'18, Week 10

z-transform: $X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$

DFT: $X(k) = \sum_{n=0}^{N-1} w_N^{kn} x(n)$, where $w_N = e^{-j\frac{2\pi}{N}}$

1. (2 points) Make a list of zeros and a list of poles given this z-plane view of a system H(z). Note that the zeros have an angle of $3\pi/4$.



$z_k = 1\angle \pm \frac{3\pi}{4} = -\frac{1}{\sqrt{2}} \pm j\frac{1}{\sqrt{2}}$

$p_k = \pm \frac{1}{2} j$

2. (2 points) Given the roots you listed above, write out H(z). Fully expand the numerator and the denominator. Multiply by z^{-1}/z^{-1} as many times as needed to eliminate positive exponents.
3. (2 points) Recall that H(z) = Y(z) / X(z). Take the inverse z-transform of your result in 2 and solve for y(n) to determine the difference equation that implements the system H(z).

(2) $H(z) = \frac{Y(z)}{X(z)} = \frac{(z - 1\angle \frac{3\pi}{4})(z - 1\angle -\frac{3\pi}{4})}{(z - j/2)(z + j/2)} = \frac{z^2 + \sqrt{2}z + 1}{z^2 + 1/4} \cdot \frac{z^{-2}}{z^{-2}} = \frac{1 + z^{-1}\sqrt{2} + z^{-2}}{1 + z^{-2}\cdot\frac{1}{4}}$

(3) $Y(z)(1 + \frac{1}{4} z^{-1}) = X(z)(1 + z^{-1}\sqrt{2} + z^{-2})$

$y(n) + \frac{1}{4} y(n-2) = x(n) + \sqrt{2}x(n-1) + x(n-2)$

$y(n) = -\frac{1}{4} y(n-2) + x(n) + \sqrt{2}x(n-1) + x(n-2)$

4. (2 points) In MATLAB, x = [6 3 5 1] and h = [4 2 5 1]. y = conv(h,x) is executed and correctly gives y = [24 24 56 35 30 10]. We attempt to perform the convolution in the DFT domain, y2 = ifft(fft(h).*fft(x)). This not only gives the wrong answer, but it gives an answer of the wrong length: [54 34 57 35]. ***Correct*** the given MATLAB code for calculating y2 using the DFT.

5. (2 points) Calculate the DFT value X(1) for x = [3 4 6 7].

(4) $N_x = 4$ $N_h = 4$ $N_y = N_x + N_h - 1 = 4 + 4 - 1 = 7$

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y2 = ifft(fft(h,7) .* fft(x,7));
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Ø-pad to length 7

OR

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y2 = ifft(fft([h 0 0 0]) .* fft([x 0 0 0]));
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(5) $W_4 = e^{-j\frac{2\pi}{4}} = -j$

$k = 1$ $n = 0..3$

$W_4^{kn} = W_4^{1\cdot n} = W_4^{n} = [1 \;\; -j \;\; -1 \;\; j]$

Dot w $x = [3 \;\; 4 \;\; 6 \;\; 7]$

$\cancel{-3j - 4 + 6j + 7} = \cancel{\boxed{3+3j}}$

$3 - 4j - 6 + 7j = \boxed{-3+3j}$