Name Answers

EE-3220-11 – Dr. Durant – Quiz 9
Winter 2015-'16, Week 9

1. (2 points) A signal containing frequencies up to 3300 Hz is sampled, and a DFT is computed. If the frequency spacing of the DFT must be no greater than 2.5 Hz, what is the minimum number of samples needed? Show your work.

$f_s \geq 6600$ Hz

(-1/2) if $f_s = 3300$ Hz

$res = \frac{f_s}{N}$

$N = \frac{f_s}{res} = \frac{3300}{2.5} = 1320$ ~~samples~~ 2640

2. (3 points) The pole of a notch filter serves to cancel the zero at nearby frequencies. Depending on specifications, notch filter pole radii are typically between 0.9 and 0.995. Discuss what happens when the pole radius is

   a. Too small (e.g., 0.7)
   - notch is much too wide
   - little zero-cancellation effect at many w far from ω_0

   b. Too large (e.g., 0.999999)
   - roundoff error causes complete cancellation of the zero, removing the notch
   - OR, roundoff error results in an unstable system

   OR - very long transient

   c. Greater than 1
   - unstable system

Recall that the formula for the DFT is $X(k) = \sum_{n=0}^{N-1} w_N^{kn} x(n)$, where $w_N = e^{-j\frac{2\pi}{N}}$

3. (2 points) Calculate the 4×4 DFT matrix, recalling that *n* varies across rows and *k* varies across columns. Express values in rectangular form.

$w_4 = e^{-j\frac{2\pi}{4}} = -j$

(-1/2 conj result)

$$D = \begin{matrix} \\ 0 \\ 1 \\ 2 \\ 3 \end{matrix} \begin{matrix} \begin{matrix} 0 & 1 & 2 & 3 \end{matrix} \\ \begin{bmatrix} w_4^0 & w_4^0 & w_4^0 & w_4^0 \\ w_4^0 & w_4^1 & w_4^2 & w_4^3 \\ w_4^0 & w_4^2 & w_4^4 & w_4^6 \\ w_4^0 & w_4^3 & w_4^6 & w_4^9 \end{bmatrix} \end{matrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

4. (1 point) Apply that 4×4 matrix operator to the column vector x(n) = [-2; 2; -2; 2] to find X(k), the DFT of x(n).

$$X = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} -2 \\ 2 \\ -2 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -8 \\ 0 \end{bmatrix}$$

5. (1 point) A real FIR filter has a zero in its z-transform at $2\angle\pi/3$. Describe any additional zero(s) that H(z) must have.

conjugate: $2\angle -\frac{\pi}{3}$

6. (1 point) What additional zero(s), if any, must the filter have if it is symmetric?

reciprocals: $\frac{1}{2}\angle -\frac{\pi}{3}$ & $\frac{1}{2}\angle \frac{\pi}{3}$