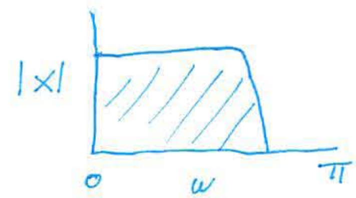
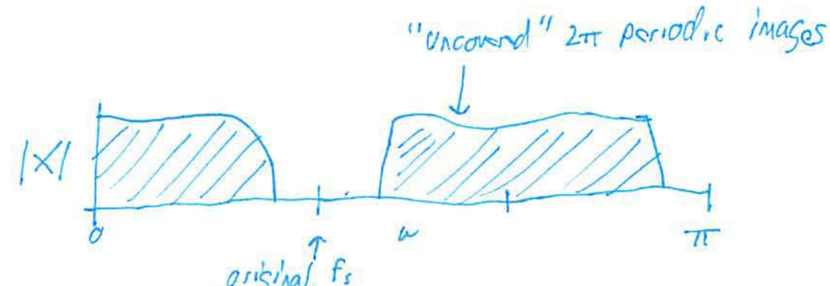
Name Answers

EE-3220-11 – Dr. Durant – Quiz 9
Winter 2014-'15, Week 9

1. (1 point) Sketch the magnitude spectrum from 0 to π radians/sample of a signal that has equal energy at all frequencies from 0 to 2π/3 and no energy at higher frequencies.

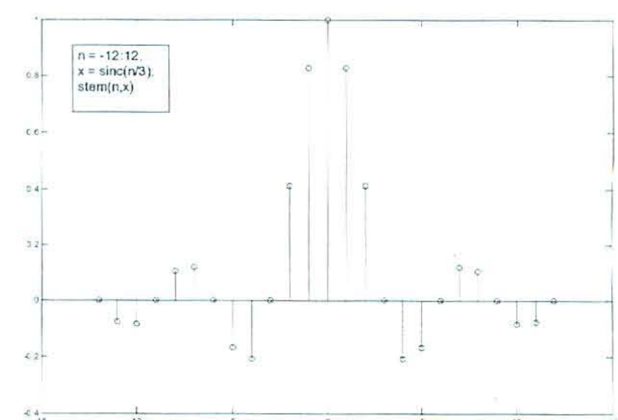


2. (1 point) Sketch the magnitude spectrum that results from increasing the sampling rate by a factor of 3 using 0 insertion (i.e., the new signal's samples consist of [x_0 0 0 x_1 0 0 x_2 0 0 x_3 ...]).



The following sinc interpolation filter will repair the spectrum of the upsampled signal:

3. (2 points) This sinc filter is not causal. Discuss the practical ramifications of this.



Unless we can do offline processing (where we know the entire signal in advance), we need to delay the output by 11 samples to make it causal and thus realizable. We cannot truncate the left half of the filter since it would do too much damage to the spectrum.

4. (1 point) The sinc filter above was truncated (in time). Discuss the practical ramifications of this.

Truncation in time does damage the spectrum, but not by a lot since there isn't much energy beyond ±12. We could ~~take~~ calculate the spectrum of the truncated sinc function to see how much it varies from ideal lowpass behavior.

5. (3 points) You have a signal with exactly 27,031 samples (a prime number) and decide to compute the spectrum with an FFT of length $2^{15} = 32{,}768$.

   a. Defend the choice of using 32,768 based on efficiency.

It is a power of 2, for which extremely fast DFT implementations are readily available. (Note: there are FFTs for other lengths, and even prime lengths, but they're not as commonly available and not quite as fast.)

   b. What is the frequency resolution in radians/sample?

$2\pi/27031$

   c. What is the frequency density in radians/sample?

$2\pi/32768 = \pi/16384$

6. (2 points) Fill in the blanks: under the bilinear transform with sampling frequency fs Hz, analog frequencies (Ω) range from 0 to _∞_ radians/second and the corresponding digital frequencies (ω) range from 0 to a maximum of _π_ radians/sample.

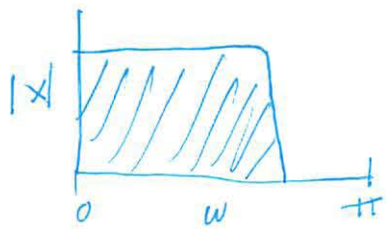
7. (Extra credit: 1 point) The slope $d\omega/d\Omega$ is _*1/fs*_ at (0,0).

And, if frequencies weren't warped, the graph of ω as a function of Ω would be a straight line from the origin to (fs/π, π), where fs/π is the maximum frequency in radians/second that is not aliased.

Name Answers

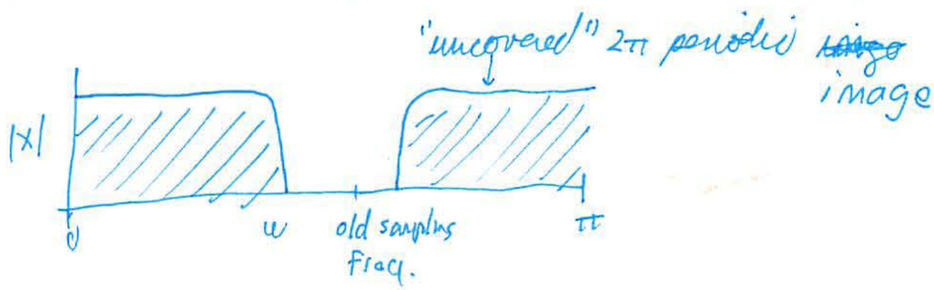
EE-3220-21 – Dr. Durant – Quiz 9
Winter 2014-'15, Week 9

1. (1 point) Sketch the magnitude spectrum from 0 to π radians/sample of a signal that has equal energy at all frequencies from 0 to π/3 and no energy at higher frequencies.

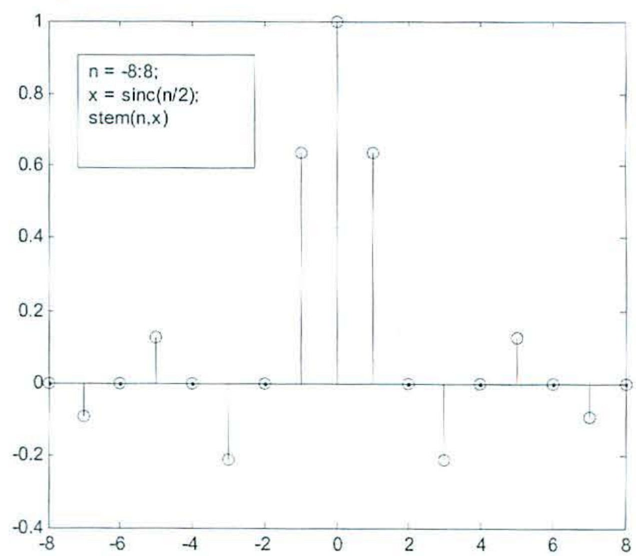


The spectrum should be half of this width; it should end at pi/3.

2. (1 point) Sketch the magnitude spectrum that results from increasing the sampling rate by a factor of 2 using 0 insertion (i.e., the new signal's samples consist of [x_0 0 x_1 0 x_2 0 x_3 ...]).



The following sinc interpolation filter will repair the spectrum of the upsampled signal:



3. (2 points) This sinc filter is not causal. Discuss the practical ramifications of this.

Unless we can do offline processing (where we know the entire signal in advance), we need to delay the output by 11 samples to make it causal and thus realizable. We cannot truncate the left half of the filter since it would do too much damage to the spectrum.

4. (1 point) The sinc filter is 0 at every even n except 0. Explain why this must be so for proper interpolation.

Because if it were not 0 at those values it would change the known correct sampled values from the originally sampled signal. h(0) = 1 represents that the interpolated signal passes through the interpolating points.

5. (3 points) You have a signal with exactly 27,031 samples (a prime number) and decide to compute the spectrum with an FFT of length $2^{15} = 32{,}768$.
    a. Defend the choice of using 32,768 based on efficiency.

It is a power of 2, for which extremely fast DFT implementations are readily available. (Note: there are FFTs for other lengths, and even prime lengths, but they're not as commonly available and not quite as fast.)

    b. Explain why the signal should be 0-padded (not partially repeated) before taking the longer DFT

0-padding doesn't change the underlying DTFT that we're sampling, just the points were sampling it at. If we repeated part of the signal, we'd corrupt the spectrum just as truncating the signals partway through a period corrupted the spectra in lab 7.

    c. Describe how the view of the spectrum would change if the signal were 0-padded to a much greater length (e.g., 2^{20})

We'd have a much smoother view interpolating the samples of the DTFT. This may be convenient for viewing, but it doesn't represent any additional information.

6. (2 points) Fill in the blanks: under the bilinear transform with sampling frequency fs Hz, a filter will have its kneepoint frequency shifted to a _***lower***_ (lower or higher?) digital frequency than designed. This transformation that shifts the analog frequency in the other direction so that the resulting digital kneepoint frequency is correct is called _***frequency warping***_.

7. (Extra credit: 1 point) When applying the bilinear transformation where do poles in the left half of the s-plane appear in the z-plane?

Inside the unit circle.