Name Answers

EE-3221-11 – Dr. Durant – Quiz 8
Winter 2017-'18, Week 8

z-transform: $X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$

DFT: $X(k) = \sum_{n=0}^{N-1} w_N^{kn} x(n)$, where $w_N = e^{-j\frac{2\pi}{N}}$

1. (2 points) A causal, stable, real-coefficient, difference equation is of second order in both x and y (that means y(n-2) and x(n-2) are used, but not older samples). The ***roots*** of the transfer function include $2\angle\pi/3$ and $0.8\angle 2\pi/3$. List all of the ***zeros*** of the transfer function.
2. (2 points) In a DFT, X(7) = 5 and N = 20. Give a complete list of k values on (-∞,∞) for which X(k) must be 5. Assume that x(n) is a real signal (which makes the DFT conjugate-symmetric).
3. (2 points) An analog signal is to be sampled at 96 kHz. A frequency resolution of 0.5 Hz (or better) is required. What is the minimum length of the DFT that should be used? What amount of time does this represent?
4. (2 points) Calculate the 4×4 DFT matrix, recalling that *k* varies across rows and *n* varies across columns. Express values in rectangular form.
5. (2 points) Use the matrix to calculate the DFT of the signal at the Nyquist frequency: ~~[0 1 0 1]~~ -1 1 -1 1.

① zeros = $2\angle\pm\pi/3$ ↑ 2 zeros. (These can't be the poles since $|p_k|<1$ for stability. Must have conjugate roots for real a, b coeffs)

② DTFT period = 2π. DFT fundamental = $\frac{2\pi}{N}$ ∴ DFT period = N.
Also, $X(-7) = 5$ since real signals have conj. symmetry.
So, we get $X(k)=5$ @ $k = 7+20l, -7+20l$ for all integers l.

$k = \ldots, -27, -13, -7, 7, 13, 27, 33, 47, \ldots$

③ $\frac{2\pi}{N}$ = ω resolution → $f_r = \frac{2\pi}{N}\cdot\frac{f_s}{2\pi} = \frac{f_s}{N}$ → $0.5 = \frac{96k}{N}$ → $N = 192{,}000$

$T = N/f_s = 192000/(96000\,\text{Hz}) = 2\,\text{s}$ OR: $T = \frac{1}{f_{res}} = \frac{1}{0.5\,\text{Hz}} = 2\,\text{s}$

④ $W_4 = e^{-j\frac{2\pi}{4}} = -j$ Using given formula: $D = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$ ← w_4 to powers 0,1,2,3

⑤ $X = Dx = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}\begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -4 \\ 0 \end{bmatrix}$ ← $k=2$ ∴ $\omega = \frac{2\pi}{N}\cdot k = \frac{2\pi}{4}\cdot 2 = \pi$ ↑ Nyquist, as expected!

Name *Answers*

EE-3221-41 – Dr. Durant – Quiz 8
Winter 2017-'18, Week 8

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z-transform: $X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$ DFT: $X(k) = \sum_{n=0}^{N-1} w_N^{kn} x(n)$, where $w_N = e^{-j\frac{2\pi}{N}}$

1. (2 points) In a DFT, X(3) = 5 and N = 30. Give a complete list of k values for which X(k) must be 30. You *cannot* assume that x(n) is a real signal (which would give conjugate symmetry of the DFT).
2. (2 points) An analog signal is sampled at 48 kHz. A 256-point DFT is computed. What is the resolution of the DFT in hertz?
3. (2 points) A signal is 0-padded to double its original length. Which (if any) of the following change as a result of the 0-padding? z-transform, DTFT, DFT
4. (2 points) Calculate the 4×4 DFT matrix, recalling that *k* varies across rows and *n* varies across columns. Express values in rectangular form.
5. (2 points) Use the matrix to calculate the DFT of ~~the~~ this signal at the ~~Nyquist~~ $\frac{\pi}{2}$ frequency: [0 -1 0 1].

① DTFT has period 2π. DFT freq. step is $\frac{2\pi}{N}$, therefore DFT has period N. $k = 3 + l \cdot 30$ for all integers l.

$k = \ldots, -57, -27, 3, 33, 63, \ldots$

② $\omega_R = \frac{2\pi}{N}$ $F_R = \frac{\omega_R}{2\pi} \cdot f_s = \frac{f_s}{N} = \frac{48\,kHz}{256} = \frac{3}{16}\,kHz = 187.5\,Hz$

③ Only the DFT changes. It doubles in length, sampling the DTFT twice as often on $[0, 2\pi)$.

④ Using given formula @ top: $D = \begin{bmatrix} W_N^0 & W_N^0 & W_N^0 & W_N^0 \\ W_N^0 & W_N^1 & W_N^2 & W_N^3 \\ W_N^0 & W_N^2 & W_N^4 & W_N^6 \\ W_N^0 & W_N^3 & W_N^6 & W_N^9 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$

$W_N = W_4 = e^{-j\frac{2\pi}{4}} = 1\angle -\frac{\pi}{2} = -j$

⑤ $X = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 0 \\ -1 \\ 0 \\ +1 \end{bmatrix} = \begin{bmatrix} -1+1 \\ j+j \\ 1-1 \\ -j-j \end{bmatrix} = \begin{bmatrix} 0 \\ 2j \\ 0 \\ -2j \end{bmatrix}$ ← No DC energy

← No Nyquist Energy

← Conjugate "−k"

~~Nyquist~~ → $\omega = \pi/2$ DFT: $\omega = \frac{2\pi}{N} \cdot k$ Together: $\frac{\pi}{2} = \frac{2\pi}{4} \cdot k$, → $k = 1$

Given x