Name *Answers*

EE-3220 – Dr. Durant – Quiz 9
Winter 2016-'17, Week 9

1. (2 points) A signal containing frequencies up to 3300 Hz is sampled, and a DFT is computed. If the frequency spacing of the DFT must be no greater than 5 Hz, what is the minimum number of samples needed? Show your work.
2. (1 point) A ***real*** FIR filter has a zero in its z-transform at $1.5\angle\pi/4$. List any additional zero(s) that H(z) must have.
3. (1 point) What ***additional*** zero(s), if any, must the filter have if h(n) is ***symmetric***?

(1) $f_s \geq 2f_{max} = 2\cdot 3300\text{ Hz} = 6600\text{ Hz}$

$f_s = 6600\text{ Hz}$ since we want minimum samples

$F_{res} = \frac{F_s}{N} \rightarrow N = \frac{f_s}{F_{res}} = \frac{6600}{5} = 1320$

(2) Conjugate: $1.5\angle -\pi/4$

(3) Reciprocal: $\frac{2}{3}\angle -\frac{\pi}{4}$ & $\frac{2}{3}\angle +\frac{\pi}{4}$

Recall that the formula for the DFT is $X(k) = \sum_{n=0}^{N-1} w_N^{kn} x(n)$, where $w_N = e^{-j\frac{2\pi}{N}}$

4. (2 points) Calculate the 4×4 DFT matrix, recalling that n varies across rows and k varies across columns. Express values in rectangular form.
5. (1 point) Let x(n) be a 4-sample sequence sinewave with no phase shift, amplitude 2, and ω = π/2. Calculate the samples of x(n).
6. (1 point) Apply that 4×4 matrix operator to the column vector x(n) = ~~[0; 2; 0; -2]~~ to find X(k), the DFT of x(n).
7. (2 points) Given the definition of x(n), explain which bin(s) (k value(s)) of its DFT you expect to be non-0. If you did everything above correctly, this will agree with your previous answer.

(4) $w_4 = e^{-j\frac{2\pi}{4}} = e^{-j\frac{\pi}{2}} = 1\angle -\frac{\pi}{2} = 1\angle -90° = \boxed{-j}$

$$D_4 = \begin{bmatrix} (-j)^0 & (-j)^0 & (-j)^0 & (-j)^0 \\ (-j)^0 & (-j)^1 & (-j)^2 & (-j)^3 \\ (-j)^0 & (-j)^2 & (-j)^4 & (-j)^6 \\ (-j)^0 & (-j)^3 & (-j)^6 & (-j)^9 \end{bmatrix} = \boxed{\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}}$$

(5) $x(n) = 2\sin\left(\frac{\pi}{2}n\right) = \ldots \boxed{[0 \quad 2 \quad 0 \quad -2]}$

n	$\frac{\pi}{2}n$	$\sin\left(\frac{\pi}{2}n\right)$
0	0	0
1	$\pi/2$	1
2	π	0
3	$3\pi/2$	-1

(6) $$X = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 0 \\ -2 \end{bmatrix} = \begin{bmatrix} 2-2 \\ -2j-2j \\ -2-(-2) \\ 2j-(-2j) \end{bmatrix} = \begin{bmatrix} 0 \\ -4j \\ 0 \\ 4j \end{bmatrix}$$

(7) $\omega = \frac{\pi}{2} = \frac{2\pi}{4} \cdot k \rightarrow k = \frac{\pi}{2} \cdot \frac{4}{2\pi} = \boxed{1}$

N

From conj. symmetry, energy also @ $\boxed{N} - \boxed{k} = \boxed{3}$

period in freq, period N

conj @ -freq

Note: As shown, (6) + (7) agree