

EE-3220-11 - Dr. Durant - Quiz 8
 Winter 2015-'16, Week 8

1. (1 point) What is the relationship between the DTFT and the DFT? (Hint: Consider the domain where each is defined.)

The DTFT is a function of the continuous variable ω .

The DFT samples the DTFT at N equally spaced frequencies starting at 0 and increasing by $\frac{2\pi}{N}$. (Thus the ~~max~~ maximum DFT frequency is $2\pi \frac{N-1}{N}$, just under 2π .)

2. (3 points) The DFT $X(k) = [12 \ 3+3j \ 0 \ 0 \ 0 \ 3-3j]$ for a 6-sample signal.

- a. (1 point) How do you know that the signal $x(n)$ is real valued?

$X(k)$ has conjugate symmetry
 $X(1) = X^*(5) = X^*(5-6) = X^*(-1)$
 \uparrow
 2π -periodic

- b. (1 point) What is the DC component's level in $x(n)$?

$$X(0) = 12 = \sum_{n=0}^5 x(n) \cdot e^{j0} = \sum x$$

$(-1/4)$ if 12

$$DC = \frac{1}{N} \sum x = \frac{1}{6} \cdot 12 = \boxed{2}$$

- c. (1 point) What is the frequency of the non-DC component? Give both k and ω .

$$k=1$$

$$\omega = \frac{2\pi}{N} \cdot k = \frac{2\pi}{6} \cdot 1 = \boxed{\frac{\pi}{3}}$$

3. (1 point) Calculate w_{12} , the 12th root of unity that represents the minimum magnitude negative angle phase shift in a 12-point DFT. Give your answer in polar form with the angle expressed as a multiple of π .

$$w_{12} = e^{-j \frac{2\pi}{N}} = e^{-j \frac{2\pi}{12}} = e^{-j \frac{\pi}{6}} = \boxed{1 \angle -\frac{\pi}{6}}$$

$(-1/4)$

4. (1 point) An analog signal is sampled at 48 kHz. A 64-point DFT is computed. What is the resolution of the DFT in hertz?

$$res = \frac{f_s}{N} = \frac{48 \text{ kHz}}{64} = 750 \text{ Hz}$$

(-1/2) not in hertz

5. (2 points) The 64-point sample above 0-padded to 256 samples and then a 256-point DFT is computed. State both the spectral resolution and spectral density of the result.

resolution is unchanged, $\boxed{750 \text{ Hz}}$

$$density = \frac{f_s}{N^*} = \frac{48 \text{ kHz}}{256} = \boxed{187.5 \text{ Hz}}$$

(-1/2) density only
(-1/4) calculations/simplify

6. (2 points) In MATLAB, $x = [4 \ 5 \ -3 \ 6]$ and $h = [2 \ -3 \ -1 \ 2]$. $y = \text{conv}(h, x)$ is executed and correctly gives $y = [8 \ -2 \ -25 \ 24 \ -5 \ -12 \ 12]$. We attempt to perform the convolution in the DFT domain, $y_2 = \text{ifft}(\text{fft}(h) \cdot \text{fft}(x))$, but get the circular convolution result instead. Calculate the values contained in y_2 .

$$\begin{array}{cccc} 8 & -2 & -25 & 24 \\ + & -5 & -12 & 12 \\ \hline \end{array} \quad \text{wraparound after } N=4 \text{ samples}$$

$$y_2 = \begin{bmatrix} 3 & -14 & -13 & 24 \\ \uparrow & & & \end{bmatrix}$$

(-1/2) erroneous flip of $x(4:6)$ or similar