Name *Answers*

EE-3220-11 – Dr. Durant – Quiz 8
Winter 2014-'15, Week 8

1. (3 points) The pole of a notch filter serves to cancel the zero at nearby frequencies. Depending on specifications, notch filter pole radii are typically between 0.9 and 0.995. Discuss what happens when the pole radius is

   a. Too small (e.g., 0.7)

   The notch is excessively wide.

   b. Too large (e.g., 0.999999) (also true that ∢ nonlinearity is greater)

   The notch is too narrow
   OR The transient response is too long
   OR round-off cancellation (unlikely, but could happen dep. on precision)

   c. Greater than 1

   The filter has an unstable response

2. (1 point) Describe the phase response of a symmetric FIR filter. Be complete for full credit.

   It is linear ~~and represents~~ and reflects a constant delay of order/2 samples.
   (3/4) (1/4)
   OR
   It is fully described by the term $e^{-j\omega\frac{M-1}{2}}$, where M is filter length.

3. (1 point) Describe the constraints on the zero locations in a symmetric FIR filter.

   Their reciprocals must also be zero.

   MORE: Real coefficients further require that the conjugate zeros exist, forming constellations of 4 zeros in the general case.

   (-1/2) "must be on unit circle/w/ conj." ← this is true for all real→real systems

Recall that the formula for the inverse DFT is $x(n) = \frac{1}{N}\sum_{k=1}^{N-1} \overset{-}{w}{}_N^{kn} X(k)$, where $w_N = e^{-j\frac{2\pi}{N}}$

4. (2 points) Calculate the 4×4 IDFT matrix, recalling that *n* varies across rows and *k* varies across columns. Express values in rectangular form.

$N=4$

$W_4 = e^{-j\frac{2\pi}{4}} = -j$

$$D^{-1} = \frac{1}{N}\begin{bmatrix} w_4^0 & w_4^0 & w_4^0 & w_4^0 \\ w_4^0 & w_4^{-1} & w_4^{-2} & w_4^{-3} \\ w_4^0 & w_4^{-2} & w_4^{-4} & w_4^{-6} \\ w_4^0 & w_4^{-3} & w_4^{-6} & w_4^{-9} \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & +j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & +j \end{bmatrix}$$

5. (2 point) Apply that 4×4 matrix operator to the column vector X(k) = [24; 4-4j; 0; 4~~-~~+4j] to find x(n), the IDFT of X(k).

$$x(n) = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & +j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & +j \end{bmatrix}\begin{bmatrix} 24 \\ 4-4j \\ 0 \\ 4+4j \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 24+4+4 \\ 24+\cancel{4j}+\bar{4}j^2+\cancel{\bar{4}j}+\bar{4}j^2 \\ 24-4+\cancel{4j}-4-\cancel{4j} \\ 24+\cancel{\bar{4}j}+4j^2+\cancel{4j}+4j^2 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 32 \\ 32 \\ 16 \\ 16 \end{bmatrix} = \begin{bmatrix} 8 \\ 8 \\ 4 \\ 4 \end{bmatrix}$$

6. (1 points) What constraints are there on X(k) when x(n) is real? Be complete and unambiguous for full credit.

X(k) is conjugate symmetric ← sufficient answer

$X(-k) = \boxed{X(N-k) = X^*(k)}$ ← preferred answer

apply period N

best answer: shows that $X(0) \in \mathbb{R}$
and if N is even, $X(\frac{N}{2}) \in \mathbb{R}$

Name *Answers*

EE-3220-21 – Dr. Durant – Quiz 8
Winter 2014-'15, Week 8

1. (3 points) The pole of a notch filter serves to cancel the zero at nearby frequencies. Depending on specifications, notch filter pole radii are typically between 0.9 and 0.995. Discuss what happens when the pole radius is
   a. Too small (e.g., 0.7)

   The notch is too wide.

   b. Too large (e.g., 0.999999)

   The notch is too narrow.
   OR The transient response is too long

   (OK answers: - round-off cancellation - greater non-linearity)

   c. Greater than 1

   The filter has an unstable response.

2. (1 point) Describe the phase response of a symmetric FIR filter. Be complete for full credit.

   It is linear, reflecting a constant delay of order/2 samples.
   (3/4) (+1/4)
   OR
   It is fully described by the term $e^{-j\omega\frac{M-1}{2}}$, where M is the filter length.

3. (1 point) Describe the constraints on the zero locations in a symmetric FIR filter.

   Their reciprocals must also be zeros.

   MORE: Real coefficients further require that the conjugate zeros exist, forming constellations of 4 zeros in the general case.
   (-1/2) if just conj. (note: being on unit circle is not required)

Recall that the formula for the DFT is $X(k) = \sum_{n=1}^{N-1} w_N^{kn} x(n)$, where $w_N = e^{-j\frac{2\pi}{N}}$

4. (2 points) Calculate the 4×4 DFT matrix, recalling that *n* varies across rows and *k* varies across columns. Express values in rectangular form.

$N=4$

$w_4 = e^{-j\frac{2\pi}{4}} = -j$

$$D = \begin{bmatrix} w_4^0 & w_4^0 & w_4^0 & w_4^0 \\ w_4^0 & w_4^1 & w_4^2 & w_4^3 \\ w_4^0 & w_4^2 & w_4^4 & w_4^6 \\ w_4^0 & w_4^3 & w_4^6 & w_4^9 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

5. (2 point) Apply that 4×4 matrix operator to the column vector x(n) = [1; 2; 3; 4] to find X(k), the DFT of x(n).

$$X(k) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 1+2+3+4 \\ 1-2j-3+4j \\ 1-2+3-4 \\ 1+2j-3-4j \end{bmatrix} = \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

6. (1 points) What constraints are there on X(k) when x(n) is real? Be complete and unambiguous for full credit.

$X(k)$ is conjugate symmetric. ← sufficient answer

$\underset{\uparrow \text{ apply period } N}{X(-k) =} \boxed{X(N-k) = X^*(k)}$ ← preferred answer

best answer adds: $X(0) \in \mathbb{R}$

+, if N is even, $X(\frac{N}{2}) \in \mathbb{R}$