Name *Answers*

EE-3220-11 – Dr. Durant – Quiz 8
Spring 2015, Week 8

1. (1 point) In MATLAB, x = [5 3 7 2] and h = [-3 2 -1 1]. y = conv(h,x) is executed and correctly gives y = [-15 1 -20 10 0 5 2]. We attempt to perform the convolution in the DFT domain, y2 = ifft(fft(h).*fft(x)), but get the circular convolution result instead. Calculate the values contained in y2.

   $N=4$

   -15 1 -20 10 0 5 2

   0 5 2

   $y2 = \boxed{-15 \quad 6 \quad \cancel{0} \quad 10}$

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2. (2 points) A signal containing frequencies up to 2500 Hz is sampled, and a DFT is computed. If the frequency spacing of the DFT must be no greater than 0.2 Hz, what is the minimum number of samples needed?

   duration: $T = \frac{1}{F} = \frac{1}{0.2\,Hz} = 5\,s$ $\quad N = T \cdot f_s = 5\,s \cdot 5000 \frac{\text{samples}}{s} = 25000$ samples

   2x F_{max} no aliasing

   Alternate: $f_s = 2 \times f_{MAX} = 5000\,Hz$ $\quad res = \frac{2\pi}{N}$, $res_{Hz} = \frac{\omega \cdot f_s}{2\pi} = \frac{2\pi f_s}{2\pi N} = \frac{f_s}{N} = 0.2\,Hz$

   From $\omega = F/f_s 2\pi$ $\quad N = \frac{f_s}{0.2\,Hz} = \frac{5000}{0.2} = 25000$ samples

3. (3 points) The pole of a notch filter serves to cancel the zero at nearby frequencies. Depending on specifications, notch filter pole radii are typically between 0.9 and 0.995. Discuss what happens when the pole radius is

   a. Too small (e.g., 0.7)

   wide notch

   b. Too large (e.g., 0.999999)

   - notch too narrow to be useful
   - transient response too long
   - unstable if slight roundoff in HW puts pole outside unit circle

   c. Greater than 1

   - unstable

Recall that the formula for the DFT is $X(k) = \sum_{n=1}^{N-1} w_N^{kn} x(n)$, where $w_N = e^{-j\frac{2\pi}{N}}$

4. (2 points) Calculate the 4×4 DFT matrix, recalling that *n* varies across rows and *k* varies across columns. Express values in rectangular form.

$$W_4 = e^{-j\frac{2\pi}{4}} = -j$$

$$D = \begin{bmatrix} w^0 & w^0 & w^0 & w^0 \\ w^0 & w^1 & w^2 & w^3 \\ w^0 & w^2 & w^4 & w^6 \\ w^0 & w^3 & w^6 & w^9 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

5. (2 point) Apply that 4×4 matrix operator to the column vector x(n) = [2; 3; 2; 1] to find X(k), the DFT of x(n).

$$X = Dx = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2+3+2+1 \\ 2-3j-2+j \\ 2-3+2-1 \\ 2+3j-2-j \end{bmatrix} = \boxed{\begin{bmatrix} 8 \\ -2j \\ 0 \\ 2j \end{bmatrix}} \leftarrow$$

Note: $x = \begin{bmatrix} 2 \\ 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 2 \\ 2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}$

↑ $k=0$ component

$X(0) = N \cdot A = 4 \cdot 2 = 8$

$k = \pm 1$ component ($k=-1 \rightarrow N-1 = 3$)

$|X(2)| = \frac{N \cdot A}{2} = \frac{4 \cdot 1}{2} = 2$

↑ split energy bet. 2 k's

$\angle X(2) = \frac{-\pi}{2} = -90^\circ$ ∴ cos delayed by 90° = sin ↓