```matlab
%% EE3221 - Dr. Durant - Quiz 7
% Winter 2020-‘21, Week 8
% “Take-home” quiz due by end of week.
% This is an open-book quiz. Open notes. You may use a calculator. You should use MATLAB.
% Given the difference equation: y(n) = y(n-1) + 0.2 x(n) – 0.2 x(n-5)

close all
clearvars

%% 1. Find an expression for and plot (hint: freqz) the system frequency response H(ejΩ).
a = [1 -1]; % move y(n-1) to LHS
b = [1 0 0 0 0 -1]/5;
figure
freqz(b,a)
title('Problem 1')

%% 2. Find and plot the pole and zero locations (hint: zplane) for the system.
po = roots(a)
ze = roots(b)
figure
zplane(ze,po)
title('Problem 2')
% You could call zplane(b,a) directly, but the above method saves the roots in variables.
% zplane arguments as columns are roots; in rows they represent coefficients
% Note also that the the zeros from b are solutions of z^5 = 1.
% So, z = 1 angle (2π k / 5). k=-2..2 (clearly conjugate pairs) or 0..4
% will yield the solutions.
% In general, z^M - 1 has M roots on the unit circle.

%% 3. Find the final value of the step response.
% Method 1: Apply the difference equation and observe the result.
u = ones(1,10);
s = filter(b,a,u);
figure
plot(0:length(s)-1,s,'bo-')
title('Problem 3: Step Response')
% We observe that s(n) is reaches 1 at n=4 and then stays there.

% Method 2: z-transform of transfer function.
% H(z) = Y(z) / X(z) = (z^5-1)/(5(z-1))
% In problem 2, we saw that the numerator's roots include z=1, which is also
% a denominator root. So, the denominator has no uncanceled roots, leaving
% a pure MA system. Perform polynomial division
b2 = deconv(b,a); % [1 1 1 1 1]/5, a moving average filter
% The DC response of a moving average filter is
s_fv2 = sum(b2); % 1

% Method 3: Transfer function approach.
% The step response will equal the DC response, except for the transient
% response. Since the system is stable (no uncanceled pole magnitudes >=
% 1), the transient decays as n grows. Therefore the final value of the
% step response is the DC gain, which is found by letting Omega = 0 and
% letting z = exp(j*Omega) = 1. So, find H(1)
s_fv3 = polyval(b2,1); % also 1. in general polyval(b,1)/polyval(a,1), but
% that gives 0/0 here, so L'Hopital's rule would be needed.

%% 4. Evaluate your system response expression from above to find the gain
% and phase shift for sinusoids input at the following frequencies in
% radians/sample: 0, 0.4π, π. Comment on whether this agrees with your frequency response plot.
Omega = [0 0.4 1] * pi;
z = exp(1j*Omega);
H = polyval(b,z) ./ polyval(a,z); % evaluate H(z) at all 3 zs
% Result is NaN, 0, 0.2
% 0: NaN means not a number since H(exp(1j*0)) = 0/0, but 1 in the limit.
%    We already found that this was 1 in the previous problem.  This agrees
%    with 0 dB and no phase shift on the frequency response plot.
% 0.4: 0 agrees with the notch (-inf dB) at a normalized frequency of 0.4
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%      on the frequency response plot. There is a phase jump of π here since we
%      are moving through a zero the complex plane, specifically (z-exp(j2π/5))
%      is a numerator factor that goes to 0 at this frequency.
% 1: 0.2: This corresponds to -13.9794 dB and it is real, so the phase
%    shift must be 0 (or a multiple of 2π), which agrees with the plot from
%    problem 1.

%% 5. Bonus: Can you rewrite the given ARMA equation as a pure MA equation?
% Hint: the z-transform and polynomial division can help.
% Yes, above in problem 3 method 2, we solved this.
% y(n) = \sum_{k=0}^{4}(x(n-k)/5)
% y(n) = (x(n) + x(n-1) + x(n-2) + x(n-3) + x(n-4)) / 5
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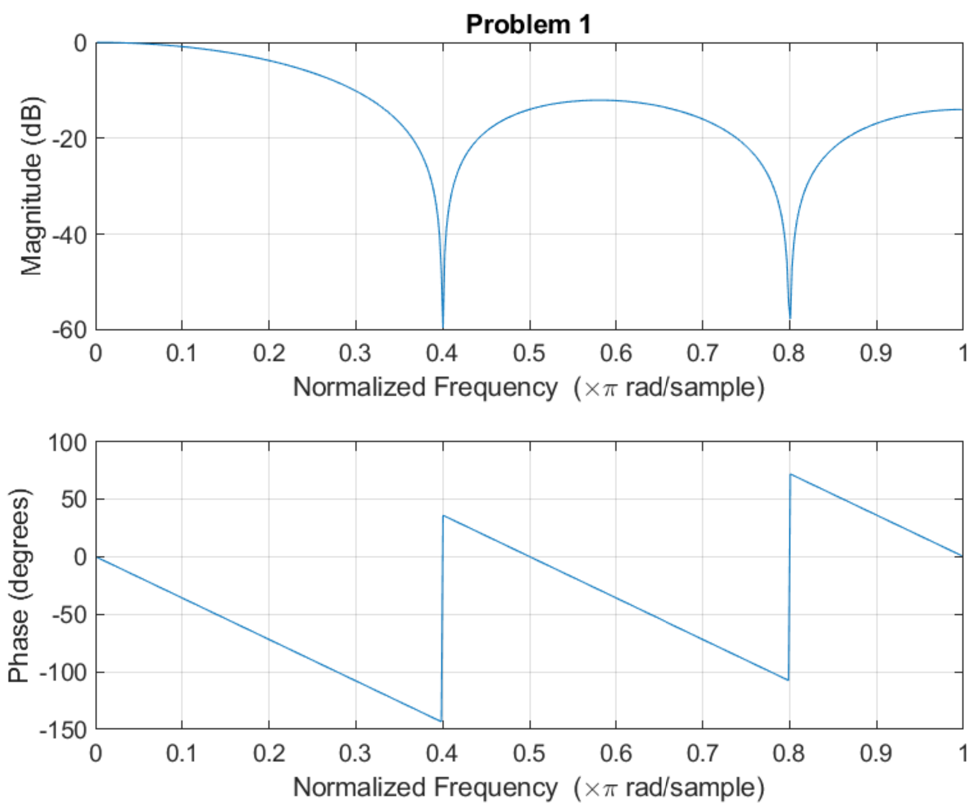
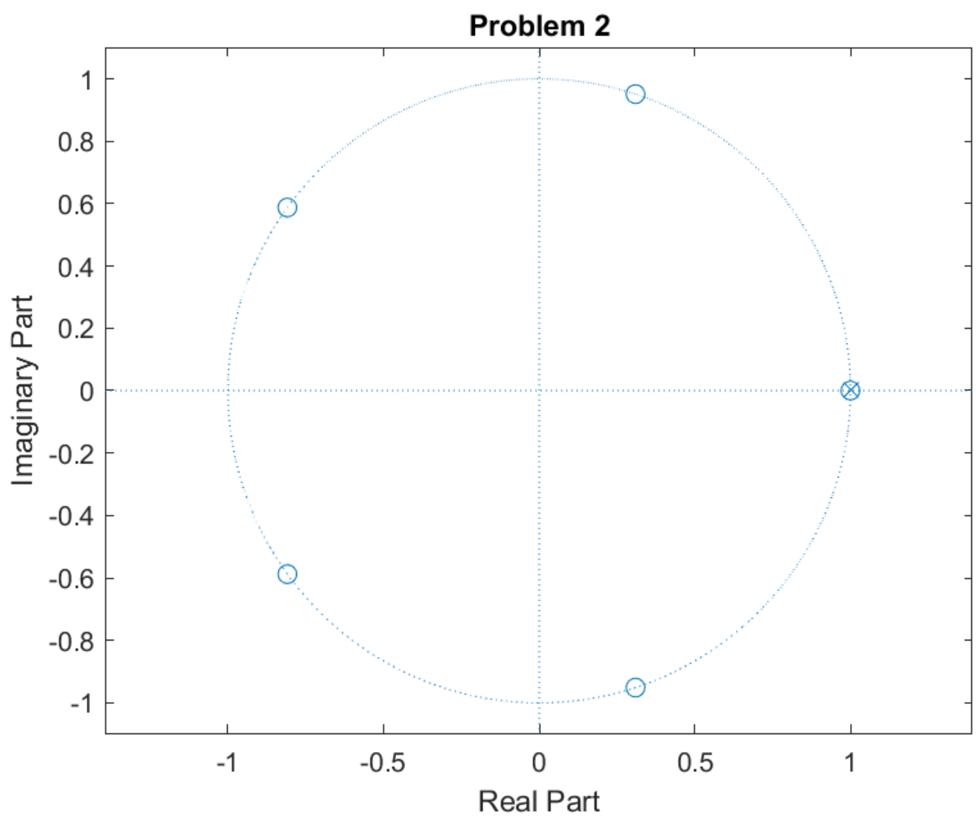
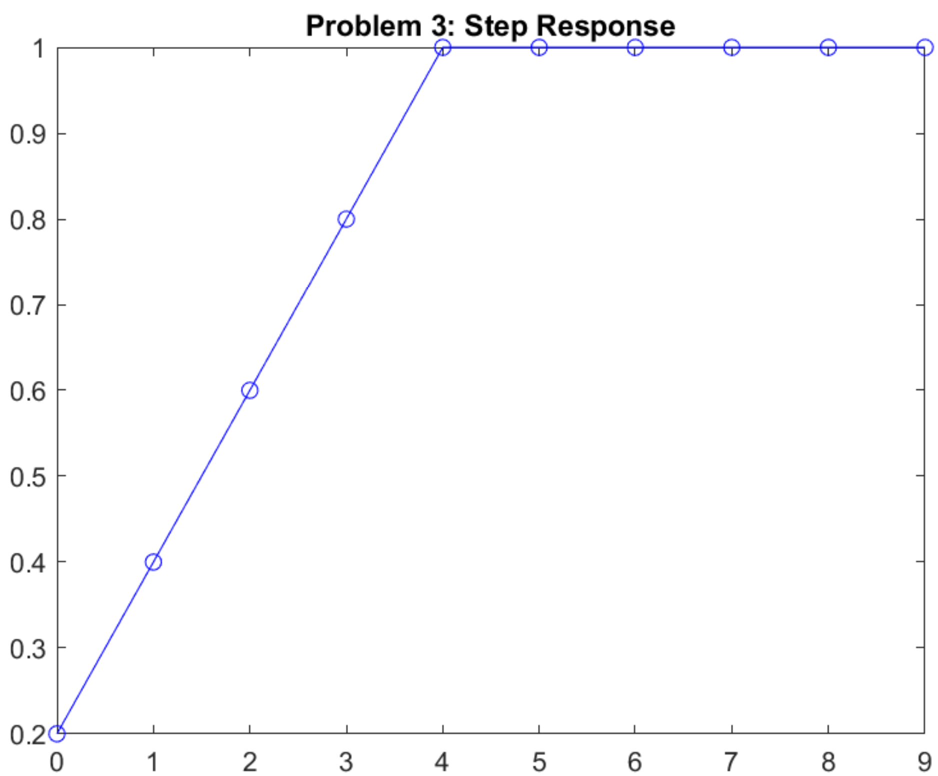


Problem 1
Magnitude (dB)
0
-20
-40
-60
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
Normalized Frequency (×π rad/sample)
Phase (degrees)
100
50
0
-50
-100
-150
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
Normalized Frequency (×π rad/sample)



Problem 2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
Imaginary Part
-1
-0.5
0
0.5
1
Real Part


Problem 3: Step Response
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0
1
2
3
4
5
6
7
8
9