Name *Answers*

EE-3221-11 – Dr. Durant – Quiz 7
Winter 2017-‘18, Week 7

Convolution: $y(n) = \sum_{k=-\infty}^{\infty} h(k)x(n-k) \leftrightarrow Y(z) = H(z)\,X(z)$ z-transform: $X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n}$

1. (4 points) Find the difference equation corresponding to H(z) = $(z^2+1.8z+1) / (z^2+1.5z+0.7)$. Solve for y(n), ensuring your result is causal.
2. (2 points) Find the zeros of H(z). Show your work. State your answer in polar form with the angle in radians and as a real number times π.
3. (4 points) Find the gain and phase shift of the system at ω’ = 0.87π. Show your work. Recall that the DTFT can be found from the z-transform by letting $z = e^{j\omega}$.

(1)
$$\frac{Y(z)}{X(z)} = \frac{1+1.8z^{-1}+z^{-2}}{1+1.5z^{-1}+0.7z^{-2}}$$
$$Y(z)(1+1.5z^{-1}+0.7z^{-2}) = X(z)(1+1.8z^{-1}+z^{-2})$$
$$y(n)+1.5y(n-1)+0.7y(n-2) = x(n)+1.8x(n-1)+x(n-2)$$
$$y(n) = -1.5y(n-1)-0.7y(n-2)+x(n)+1.8x(n-1)+x(n-2)$$

(2) $z^2+1.8z+1=0$
$$z = \frac{-1.8 \pm \sqrt{1.8^2-4}}{2} = \frac{-1.8 \pm \sqrt{-0.76}}{2} = -0.9 \pm \sqrt{\cancel{-0.38}\ -0.19} = -0.9 \pm j\,\cancel{0.76}\ 0.436$$
$$= 1\angle \pm 0.856\pi$$

(3) Note ω' is very close to the angle of the zero we just found on the unit circle, so we expect a small answer.

$$H(e^{j\omega'}) = \frac{(e^{j0.87\pi})^2 + 1.8e^{j0.87\pi} + 1}{(e^{j0.87\pi})^2 + 1.5e^{j0.87} + 0.7} = \frac{0.684 - j0.729 + 1.8(-0.912 + j.397) + 1}{0.684 - j0.729 + 1.5(-0.912 + j.397) + 0.7}$$

$$= \frac{0.0424 - j\,0.014}{0.160 - j\,0.134} = 0.199 + j\,0.079 = \underbrace{0.214}_{\text{gain}}\angle \underbrace{0.120\pi}_{\text{phase advance}}$$

Name *Answers*

EE-3221-41 – Dr. Durant – Quiz 7
Winter 2017-'18, Week 7

Convolution: $y(n) = \sum_{k=-\infty}^{\infty} h(k)x(n-k) \leftrightarrow Y(z) = H(z)\,X(z)$ z-transform: $X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n}$

1. (4 points) Find the difference equation corresponding to $H(z) = (z^2-1.2z+1) / (z^2-z+0.8)$. Solve for y(n), ensuring your result is causal.
2. (4 points) Find the gain and phase shift of the system at $\omega' = 0.3\pi$. Show your work. Recall that the DTFT can be found from the z-transform by letting $z = e^{j\omega}$.
3. (2 points) Based on your answer to 2, discuss whether $e^{j\omega'}$ likely closer to either a pole or a zero of the system.

(1)
$$\frac{Y(z)}{X(z)} = \frac{1-1.2z^{-1}+z^{-2}}{1-z^{-1}+0.8z^{-2}}$$

$$Y(z)(1-z^{-1}+0.8z^{-2}) = X(z)(1-1.2z^{-1}+z^{-2})$$

$$y(n)-y(n-1)+0.8y(n-2) = x(n)-1.2x(n-1)+x(n-2)$$

$$y(n) = y(n-1)-0.8y(n-2)+x(n)-1.2x(n-1)+x(n-2)$$

(2)
$$H(e^{j\omega'}) = \frac{(e^{j0.3\pi})^2-1.2e^{j0.3\pi}+1}{(e^{j0.3\pi})^2-e^{j0.3\pi}+0.8} = \frac{-0.309+j0.951-1.2(0.588+j0.809)+1}{-0.309+j0.951-(0.588+j0.809)+0.8}$$

$$= \frac{-0.015-j0.020}{-0.097+j0.142} = 0.0468+j0.138 = \underset{\text{gain}}{0.145}\angle \underset{\text{phase advance}}{0.604\pi}$$

(3) Gain small, may be near a zero

Extra: $\text{roots}(z^2-1.2z+1) = 1\angle\pm\underbrace{0.2952\pi}_{\text{close to } \omega'=0.3\pi}$