Name Answers

EE-3220 – Dr. Durant – Quiz 7
Winter 2016-'17, Week 8

1. (2 points) What is the relationship between the DTFT and the z-transform? (Hint: Consider the domain where each is defined.)

The DTFT is defined for steady state while the z-transform better represents decaying & rising signals. $z = e^{j\omega}$ for the DTFT; z is evaluated on the unit circle

2. (2 points) The DFT X(k) = [12 2j 0 -2j] for a 4-sample signal.
   a. (1 point) How do you know that the signal x(n) is real valued?

$X(1)^* = X(3)$ ← conjugate symmetry

   b. (1 point) What is the DC component's level in x(n)? (It may help to write the expression for X(0).)

$\frac{X(0)}{N} = \frac{12}{4} = \boxed{3}$

$X(0) = \sum_{n=0}^{3} x(n) e^{-j\omega \cdot n \cdot 0} = x(0) + x(1) + x(2) + x(3)$ ← k

$DC = \frac{\sum x(n)}{N} = \frac{X(0)}{N}$ ✓

3. (1 point) Calculate w_8, the 8[th] root of unity that represents the minimum magnitude negative angle phase shift in a 8-point DFT. Give your answer in both (a) polar form with the angle expressed as a multiple of π and (b) rectangular form.

(a) $w_8 = e^{-j\omega \frac{2\pi}{8}} = e^{-j\frac{\pi}{4}}$

(b) $w_8 = \boxed{\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}} = \frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}$

$24000 = 2^6 3 \cdot 5^3$

4. (1 point) An analog signal is sampled at 24 kHz. A 128-point DFT is computed. What is the resolution of the DFT in hertz?

Method 1: Freq step $= \frac{F_s}{N} = \frac{24{,}000}{128} = \frac{2^6 3 \cdot 5^3}{2^7} = \frac{3 \cdot 5^3}{2} = \boxed{187.5\,\text{Hz}} = 187\tfrac{1}{2}\,\text{Hz}$

Method 2: $\frac{1}{\text{Duration}} = \frac{1}{T} = \frac{1}{(N/F_s)} = \frac{F_s}{N} = \ldots$ same

5. (2 points) The 128-point sample above 0-padded to 512 samples and then a 512-point DFT is computed. Is it the spectral resolution or density that has changed and what is its new value?

4x better ... $\frac{187.5\,\text{Hz}}{4} = \boxed{46.875\,\text{Hz}} = 46\tfrac{7}{8}\,\text{Hz}$

6. (2 points) In MATLAB, x = [2 -4 -2 1] and h = [2 -4 4 5]. y = conv(h,x) is executed and correctly gives y = [4 -16 20 4 -32 -6 5]. We attempt to perform the convolution in the DFT domain, y2 = ifft(fft(h,6).*fft(x,6)). This not only gives the wrong answer, but it gives an answer of the wrong length. Explain what happened and calculate the result returned in y2.

The DTFT was sampled @ 6 points but y has 7 points. This is "temporal aliasing." The value at x_6 gets added to x_0 ... [9 -16 20 4 -32 -6].