

Name Answers

EE-3220-11 - Dr. Durant - Quiz 7  
Winter 2014-'15, Week 7

1. (1 point) A voice signal sampled at 10 kHz is intermittently jammed with a loud, 1 kHz tone. Begin the design an IIR notch filter to suppress this tone. What are the radii and angles of the poles and zeros? Present angles in terms of  $\pi$  (e.g.,  $0.7\pi$ ).

$$\omega_0 = \frac{f}{f_s} \cdot 2\pi = \frac{\pi}{5}$$

$$z = 1 \angle \pm \frac{\pi}{5}$$

$$p = 0.99 \angle \pm \frac{\pi}{5}$$

2. (1 point) In MATLAB,  $x = [4 \ -3 \ 2 \ -1]$  and  $h = [2 \ 1 \ 0 \ -1]$ .  $y = \text{conv}(h,x)$  is executed and correctly gives  $y = [8 \ -2 \ 1 \ -4 \ 2 \ -2 \ 1]$ . What is the value of  $y_2$  produced by attempting to perform the convolution with the DFT  $y_2 = \text{ifft}(\text{fft}(h) \cdot \text{fft}(x))$ ?

(-2) Same

circular convolution

$$\begin{array}{r} 8 \ -2 \ 1 \ -4 \\ \leftarrow 2 \ -2 \ 1 \end{array}$$

$$\boxed{10 \ -4 \ 2 \ -4}$$

3. (1 point) What is the relationship between the DTFT and the DFT? (Hint: Consider the domain where each is defined.)

The DFT samples the DTFT.

That is, the DFT evaluates the DTFT only  
at  $\omega = \frac{2\pi}{N} \cdot k$ ,  $N = \# \text{ of time samples} = \# \text{ of Freq. samples}$   
 $0 \leq k < N$

4. (2 points) An analog signal is sampled at 48 kHz. A 256-point DFT is computed. What is the resolution of the DFT in hertz?

$$\frac{48 \text{ kHz}}{256} = \frac{3}{16} \text{ kHz} = \frac{3000}{16} \text{ Hz} = \frac{375}{2} \text{ Hz} = 187.5 \text{ Hz}$$

5. (2 points) The 256-point sample above 0-padded to 512 samples and then a 512-point DFT is computed. State both the spectral resolution and spectral density of the result.

resolution is unchanged = 187.5 Hz  
 density doubled ( $\frac{512}{256}$ )  $\therefore \frac{187.5}{2} = 93.75 \text{ Hz}$

(-34) uHz?  
 (-34) overlap

6. (1 point) Calculate  $w_N^{12}$ , the 12<sup>th</sup> root of unity that represents the minimum magnitude negative angle phase shift in an 12-point DFT. Give your answer in polar form with the angle expressed as a multiple of  $\pi$ .

$$w_{12} = e^{-j \frac{2\pi}{12}} = e^{-j \frac{\pi}{6}} = \boxed{1 \angle -\frac{\pi}{6}}$$

$f_s \geq 3 \text{ kHz}$ , use 3 kHz  
 ↑

7. (2 points) A signal containing frequencies up to 1500 Hz is sampled, and a DFT is computed. If the frequency spacing of the DFT must be no greater than 0.2 Hz, what is the minimum number of samples needed?

$$\frac{3000 \text{ Hz}}{N} = 0.2 \text{ Hz}$$

$$N = 15000 \text{ samples}$$

(-1/4) 2x for no aliasing

EE-3220-21 - Dr. Durant - Quiz 7  
 Winter 2014-'15, Week 7

1. (1 point) A voice signal sampled at 12 kHz is intermittently jammed with a loud, 2 kHz tone. Begin the design an IIR notch filter to suppress this tone. What are the radii and angles of the poles and zeros? Present angles in terms of  $\pi$  (e.g.,  $0.7\pi$ ).

$$\omega_0 = \frac{2}{12} 2\pi = \frac{\pi}{3}$$

$$z = |c| \pm \frac{\pi}{3}$$

$$p = 0.99 |c| \pm \frac{\pi}{3}$$

2. (1 point) In MATLAB,  $x = [4 -3 2 -1]$  and  $h = [2 1 0 -1]$ .  $y = \text{conv}(h,x)$  is executed and correctly gives  $y = [8 -2 1 -4 2 -2 1]$ . We attempt to perform the convolution in the DFT domain,  $y2 = \text{ifft}(\text{fft}(h).\text{fft}(x))$ , but get the circular convolution result  $[10 -4 2 -4]$  instead. Correct the statement assigning to  $y2$  so that it correctly uses the DFT and inverse DFT to compute the standard, linear convolution.

$$y2 = \text{ifft}(\text{fft}(h,7).\text{fft}(x,7));$$

$\downarrow$   
 or  $[h \ 0 \ 0 \ 0]$   
 or  $[h \ \text{zeros}(1,3)], \text{etc}$

3. (1 point) What is the relationship between the DTFT and the DFT? (Hint: Consider the domain where each is defined.)

\* The DFT samples the DTFT. ( $\Leftrightarrow$  DFT is time-limited DTFT, results in infinity in freq.)

DTFT defined for  $\forall \omega \in \mathbb{R}$  (all real values)  
 DFT defined @  $\omega = \frac{2\pi}{N}k$ ,  $N$  samples (in both time & freq.)  
 $k = 0 \dots N-1$

4. (2 points) An analog signal is sampled at 16 kHz. A 128-point DFT is computed. What is the resolution of the DFT in hertz?

$$\frac{16 \text{ kHz}}{128} = \frac{1}{8} \text{ kHz} = \frac{1000}{8} \text{ Hz} = \boxed{125 \text{ Hz}}$$

OR: in rad:  $\frac{2\pi}{128}$ , to Hz  $\rightarrow \frac{2\pi}{128} \cdot \frac{f_s}{2\pi} = \frac{f_s}{128} = \dots$

5. (2 points) The 128-point sample above 0-padded to 1024 samples and then a 1024-point DFT is computed. State both the spectral resolution and spectral density of the result,

resolution is unchanged: 125 Hz

density increased by  $\frac{1024}{128} = 8 \rightarrow \frac{125}{8} \text{ Hz} = \boxed{15.625 \text{ Hz}}$

$$\begin{array}{r} 15.625 \\ 8 \overline{) 125} \\ \underline{45} \\ 50 \\ \underline{20} \\ 40 \end{array}$$

$\left(\frac{3}{4}\right)$  pts not Hz  
 $\left(\frac{3}{4}\right)$  reversed

6. (1 point) Calculate  $w_9$ , the 9<sup>th</sup> root of unity that represents the minimum magnitude negative angle phase shift in a 9-point DFT. Give your answer in polar form with the angle expressed as a multiple of  $\pi$ .

$$w_N = e^{-j \frac{2\pi}{N}}$$

$$w_9 = e^{-j \frac{2\pi}{9}} = \boxed{1 \angle -\frac{2\pi}{9}}$$

7. (2 points) A signal containing frequencies up to 2000 Hz is sampled, and a DFT is computed. If the frequency spacing of the DFT must be no greater than 0.05 Hz, what is the minimum number of samples needed?

$$f_s \geq 2f_{\text{max}} = 4000 \text{ Hz}$$

let  $f_s = 4000 \text{ Hz}$  exactly to minimize  $N$

$$\Delta f = \frac{f_s}{N} \leq 0.05 \text{ Hz}$$

$$N \geq \frac{f_s}{0.05 \text{ Hz}} = \frac{4000}{0.05} = 80,000$$

minimum is  $\boxed{80,000}$  samples

$\left(\frac{1}{4}\right)$  if forgot  $2f_{\text{max}}$