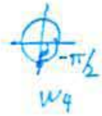
Name Answers

EE-3220-21 – Dr. Durant – Quiz 7
Winter 2013-'14, Week 9

1. (2 points) Calculate w_4, the 4[th] root of unity that represents the minimum magnitude negative angle phase shift in a 4-point DFT. Give your answer in rectangular form.

$w_N = e^{-j\frac{2\pi}{N}}$ $\quad w_4 = e^{-j\frac{\pi}{2}}$



$w_4 = \boxed{-j}$

2. (2 points) Write the general formula for the forward DFT as a summation over the sampled frequency indexes, k. Recall that w gets a positive exponent in the forward DFT.

$$X(k) = \sum_{n=0}^{N-1} x(n)\, w_N^{kn}$$

3. (2 points) Calculate the 4×4 DFT matrix based on your work above. Give your answer in rectangular form.

$$D = \begin{bmatrix} w^0 & w^0 & w^0 & w^0 \\ w^0 & w^1 & w^2 & w^3 \\ w^0 & w^2 & w^4 & w^6 \\ w^0 & w^3 & w^6 & w^9 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

4. (2 points) Apply that 4×4 matrix operator to x(n) = [-1 2 5 2] interpreted as a column vector to find X(k), the DFT of x(n). The fact that x(n) = [2 2 2 2] + [-3 0 3 0] may help you check your work.

$$X(k) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 5 \\ 2 \end{bmatrix} = \begin{bmatrix} 8 \\ -6 \\ 0 \\ -6 \end{bmatrix}$$

$\leftarrow k=0 \therefore 4\times$ DC of 2 ✓

$\leftarrow k=1,3 \therefore \frac{4}{2} = 2\times \quad \omega = \frac{2\pi k}{N} = \frac{\pi}{2}$ of -3

5. (2 points) The complexity of the Cooley-Tukey, radix-2, decimation-in-time FFT (the algorithm we derived in class), where N is a power of 2, is O(N log N). Specifically, it takes about $N \times \log_2 N$ multiplies to calculate the DFT this way. Explain the source of both the "$\log_2 N$" and "N" terms.

$\log_2 N$ = # of layers of combining half-length DFTs

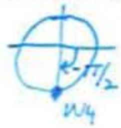
N = # of multiplies needed to get to each layer based on previous layer res

Name *Answers*

EE-3220-41 – Dr. Durant – Quiz 7
Winter 2013-'14, Week 9

1. (2 points) Calculate w_4, the 4th root of unity that represents the minimum magnitude negative angle phase shift in a 4-point DFT. Give your answer in ~~simplified~~, rectangular form.

$w_N = e^{-j\frac{2\pi}{N}}$ $\quad w_4 = e^{-j\frac{\pi}{2}}$



$w_4 = \boxed{-j}$

2. (2 points) Write the general formula for the inverse DFT as a summation over the sampled frequency indexes, k. Recall that w gets a negative exponent in the inverse DFT and that there is one other difference in its formula compared with the forward DFT.

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, w_N^{-kn}$$

3. (2 points) Calculate the 4×4 IDFT matrix based on your work above. Give your answer in ~~simplified~~, rectangular form.

$$D^{-1} = \frac{1}{4}\begin{bmatrix} w^0 & w^0 & w^0 & w^0 \\ w^0 & w^{-1} & w^{-2} & w^{-3} \\ w^0 & w^{-2} & w^{-4} & w^{-6} \\ w^0 & w^{-3} & w^{-6} & w^{-9} \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}$$

4. (2 points) Apply that 4×4 matrix operator to the column vector X(k) = [16; 8+8j; 0; 8-8j] to find x(n), the IDFT of X(k).

$$x(n) = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}\begin{bmatrix} 16 \\ 8+8j \\ 0 \\ 8-8j \end{bmatrix} = \boxed{\begin{bmatrix} \cancel{16}8 \\ \cancel{8}0 \\ 0 \\ 8 \end{bmatrix}} = \begin{bmatrix} \cancel{4}8 \\ \cancel{4}8 \\ \cancel{4}8 \\ \cancel{4}8 \end{bmatrix} + \begin{bmatrix} \cancel{8} \\ \cancel{0} \\ \cancel{-8} \\ \cancel{0} \end{bmatrix} \begin{matrix} 4 \\ -4 \\ -4 \\ 4 \end{matrix}$$

$k=0$ portion. $\quad k=1,3$ port. $\omega = \frac{2\pi k}{N} = \frac{\pi}{2}$, w/ φ shift.

5. (2 points) What constraints are there on X(k) when x(n) is real? Be complete and unambiguous for full credit.

The DTFT has conjugate symmetry: $X(e^{+j\omega}) = X^*(e^{-j\omega})$

$\therefore \boxed{X(k) = X^*(N-k)}$

For N even, $X(\frac{N}{2}) = X^*(N-\frac{N}{2}) = X^*(\frac{N}{2}) \therefore$ $\boxed{X(\tfrac{N}{2}) \text{ is real}}$

$k=0$ is DC, which is sum of real $\therefore$ real $\therefore$ $\boxed{X(0) \text{ is real}}$