Name Answers

EE-3220-11 – Dr. Durant – Quiz 7
Spring 2015, Week 7

1. (1 point) A voice signal sampled at 10 kHz is intermittently jammed with a loud, 3 kHz tone. Begin the design an IIR notch filter to suppress this tone. What are the radii and angles of the poles and zeros? Present angles in terms of π (e.g., 0.7π).

$\omega_0 = \frac{f}{f_s} 2\pi = \frac{3}{10} 2\pi = \frac{3}{5}\pi$

poles : $0.99 \angle \pm \frac{3}{5}\pi$

zeros : $1 \angle \pm \frac{3}{5}\pi$

2. (1 point) What is the relationship between the DTFT and the DFT? (Hint: Consider the domain where each is defined.)

The DFT samples the DTFT @ N equally spaced frequencies $\frac{2\pi}{N} \cdot k$ for $k = 0 \ldots N-1$

3. (3 points) The DFT X(k) = [6 0 2-2j 0 2+2j 0] for a 6-sample signal.
   a. (1 point) How do you know that the signal x(n) is real valued?

$X(k) = X^*(-k) = X^*(N-k)$ for all k

DFT is conjugate symmetric ∴ signal is real

apply frequency periodicity. $+N \rightarrow +2\pi$ in freq

since step $= \frac{2\pi}{N}$

   b. (1 point) What is the DC offset of x(n)?

$\frac{X(0)}{N} = \frac{6}{6} = 1$

   c. (1 point) What is the frequency of the non-DC component? Give both k and ω.

$k = 2$

$\omega = \frac{2\pi}{N} \cdot k = \frac{2\pi}{6} \cdot 2 = \frac{2\pi}{3}$

4. (2 points) An analog signal is sampled at 24 kHz. A 64-point DFT is computed. What is the resolution of the DFT in hertz?

$\omega_2 = \frac{2\pi}{N} = \frac{2\pi}{64} = \frac{\pi}{32}$ rad/sample

$\Omega = \omega_2 \cdot f_s = \frac{\pi}{32}\frac{\text{rad}}{\text{samp}} \cdot 24000\frac{\text{samples}}{\text{second}} = 750\pi\frac{\text{rad}}{\text{sec}}$

$f = \frac{\Omega}{2\pi} = \frac{750\pi}{2\pi} = \boxed{375\,Hz}$

Shortcut: $T = N/f_s = 64/24000 = \frac{1}{375}$ sec ← Duration

$f = \frac{1}{T} = 375\,Hz$ ← frequency resolution

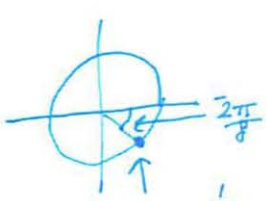
5. (2 points) The 64-point sample above 0-padded to 512 samples and then a 512-point DFT is computed. State both the spectral resolution and spectral density of the result.

Resolution is the same: 375 Hz.

Density has "improved" $375 \cdot \frac{64}{512} = 375 \cdot \frac{1}{8} = 46\frac{7}{8}$ Hz

6. (1 point) Calculate w_8, the 8th root of unity that represents the minimum magnitude negative angle phase shift in a 8-point DFT. Give your answer in polar form with the angle expressed as a multiple of π.

$w_8 = e^{-j\frac{2\pi}{8}} = e^{-j\frac{\pi}{4}} = \boxed{1\angle -\frac{\pi}{4}}$



$w_8 = \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$