

EE-3220-11 - Dr. Durant - Quiz 6
Winter 2015-'16, Week 6

Given the difference equation $y(n] = -0.4 y[n-1] + 3 x[n] - 4 x[n-1]$

1. (2 points) Take the z-transform of both sides of the equation. Remember, z^{-1} represents a sample delay.

$$Y(z) = -0.4 Y(z) z^{-1} + 3 X(z) - 4 X(z) z^{-1}$$

2. (1 point) Solve the above equation for the transfer function $H(z)$.

$$Y(z)(1 + 0.4 z^{-1}) = X(z)(3 - 4 z^{-1})$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{3 - 4 z^{-1}}{1 + 0.4 z^{-1}} = \frac{3z - 4}{z + 0.4}$$

3. (2 points) Write out the first 5 terms of $x[n]$ given that $X(z) = \frac{z}{z + 1/2}$.

$$x[n] = \left(-\frac{1}{2}\right)^n u[n] = \left\{ 1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \frac{1}{16}, \dots \right\}$$

↑

(1/2) start @ n=1
(-1/2) all neg. or pos.

4. (2 points) What will the poles of $Y(z)$ be based on $H(z)$ and $X(z)$ above? ($y[n]$ is the output of the LTI system with $h[n]$ applied to the input $x[n]$.) Explain what **each** of these poles tells you about the system response.

$$Y(z) = H(z) X(z) = \frac{3z - 4}{z + 0.4} \cdot \frac{z}{z + 0.5}$$

1 pt ID poles
1/2 pt independent contribution

poles at -0.4 and -0.5

Response has 2 decaying components, alternating signs.

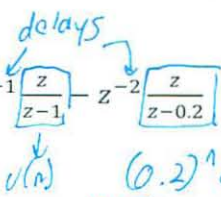
5. (1 point) Calculate the z-transform of $x = [6 \ -5 \ 2]$, which starts at $n = -2$.

$$X(z) = 6z^2 - 5z + 2$$

↑
n = -2 term

(1/2) fold ($z \leftrightarrow z^{-1}$)

6. (2 points) Calculate the inverse z-transform of $X(z) = z^{-1} \frac{z}{z-1} - z^{-2} \frac{z}{z-0.2}$



OK form

$$x[n] = u[n-1] - (0.2)^{n-2} u[n-2] = u[n-1] - \underbrace{0.2}_{0.2} (0.2)^{n-2} u[n-2] \leftarrow \text{cleaner form}$$

$$(4) Y(z) = H(z)X(z) = \frac{3z-4}{z+4} \cdot \frac{z}{z+0.5}$$

$$\frac{Y(z)}{z} = \frac{3z-4}{(z+0.4)(z+0.5)} = \frac{A}{z+0.4} + \frac{B}{z+0.5}$$

$$3z-4 = A(z+0.5) + B(z+0.4)$$

$$= (A+B)z + (0.5A+0.4B)$$

$$A+B=3$$

$$0.5A+0.4B=-4$$

$$0.5(3-B)+0.4B=-4$$

$$-0.1B = -5.5$$

$$B=55$$

$$A=-52$$

$$0.5 \cdot -52 + 0.4 \cdot 55$$

$$-26 + 22 = -4 \checkmark$$

→ solve in MATLAB

$$C = [1 \ 1; 0.5 \ 0.4];$$

$$r = [3; -4];$$

$$p = C \setminus r$$

↑ solve linear system $Cp = r$

$$\frac{Y(z)}{z} = \frac{-52}{z+0.4} + \frac{55}{z+0.5}$$

$$Y(z) = \frac{-52z}{z+0.4} + \frac{55z}{z+0.5}$$

$$y(n) = -52 \cdot (0.4)^n u(n) + 55 \cdot (-0.5)^n u(n)$$