Name Answers

EE-3220-11 – Dr. Durant – Quiz 6
Winter 2015-'16, Week 6

Given the difference equation y(n) = -0.4 y(n-1) + 3 x(n) – 4 x(n-1)

1. (2 points) Take the z-transform of both sides of the equation. Remember, z^{-1} represents a sample delay.

$$Y(z) = -0.4\,Y(z)z^{-1} + 3X(z) - 4\,X(z)z^{-1}$$

2. (1 point) Solve the above equation for the transfer function H(z).

$$Y(z)(1+0.4z^{-1}) = X(z)(3-4z^{-1})$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{3-4z^{-1}}{1+0.4z^{-1}} = \frac{3z-4}{z+0.4}$$

3. (2 points) Write out the first 5 terms of x(n) given that $X(z) = \frac{z}{z+1/2}$.

$$x(n) = \left(-\tfrac{1}{2}\right)^n u(n) = \left\{1 \quad -\tfrac{1}{2} \quad \tfrac{1}{4} \quad -\tfrac{1}{8} \quad \tfrac{1}{16} \quad \ldots\right\}$$

(-1/2) start @ n = 1
(-1/2) all neg. or pos.

4. (2 points) What will the poles of Y(z) be based on H(z) and X(z) above? (y(n) is the output of the LTI system with h(n) applied to the input x(n).) Explain what ***each*** of these poles tells you about the system response.

1 pt ID poles
1/2 pt independent contribution

$$Y(z) = H(z)X(z) = \frac{3z-4}{z+0.4} \cdot \frac{z}{z+0.5}$$

poles at -0.4 and -0.5

Response has 2 decaying components, alternating signs.

5. (1 point) Calculate the z-transform of x = [6 -5 2], which starts at ***n=-2***.

$$X(z) = 6z^2 - 5z + 2$$

$n=-2$ term

(-1/2) fold ($z \leftrightarrow z^{-1}$)

6. (2 points) Calculate the inverse z-transform of $X(z) = z^{-1}\frac{z}{z-1} - z^{-2}\frac{z}{z-0.2}$

delays

$u(n)$ $(0.2)^n u(n)$

OK form

$$x(n) = u(n-1) - (0.2)^{n-2}u(n-2) = u(n-1) - 0.2^n u(n-2)$$

$25 \cdot 0.2^2$

← cleaner form

Extra example building on #4

④ $Y(z) = H(z)X(z) = \frac{3z-4}{z+.4} \cdot \frac{z}{z+0.5}$

$$\frac{Y(z)}{z} = \frac{3z-4}{(z+0.4)(z+0.5)} = \frac{A}{z+0.4} + \frac{B}{z+0.5}$$

$$3z-4 = A(z+0.5) + B(z+0.4)$$
$$= (A+B)z + (0.5A+0.4B)$$

$A+B=3$

$0.5A+0.4B=-4$

$0.5(3-B)+0.4B=-4$

$-0.1B=-5.5$

$B=55$

$A=-52$

$0.5\cdot -52 + 0.4\cdot 55$

$-26+22=-4\checkmark$

→ solve in MATLAB

```
C=[1 1; 0.5 0.4];
r=[3; 4];
p=C\r
```

↑ solve linear system $C_p = r$

$$\frac{Y(z)}{z} = \frac{-52}{z+0.4} + \frac{55}{z+0.5}$$

$$Y(z) = \frac{-52z}{z+0.4} + \frac{55z}{z+0.5}$$

$$y(n) = -52\cdot(0.4)^n u(n) + 55\cdot(-0.5)^n u(n)$$