

EE-3221 - Dr. Durant - Quiz 5
Winter 2020-'21, Week 5

This is an **open**-book quiz. As always, you may also refer to your homework that is due today.

1. (2 points) Let $y[n] = 3x[n] - 2x[n-1] + x[n-3]$. Find the system impulse response, $h[n]$.
2. (2 points) Let $y[n] = 0.5y[n-1] + x[n]$. Find the system impulse response, $h[n]$.

① Sub in $x[n] = \delta[n]$. Then $y[n]$ becomes $h[n]$

$$h[n] = 3\delta[n] - 2\delta[n-1] + \delta[n-3]$$

$$= \{3, -2, 0, 1\}$$

~~1/2~~ (-1/2) step response

② $h[n] = \frac{1}{2}h[n-1] + \delta[n]$
 "Assume" $h[-1] = 0$. It must, since the equation is causal
 & in a known LTI form.

n	$h[n]$	$\delta[n]$	$h[n] = \frac{1}{2}h[n-1] + \delta[n]$
0	0	1	$\frac{1}{2} \cdot 0 + 1 = 1$
1	$\frac{1}{2}$	0	$\frac{1}{2} \cdot 1 + 0 = \frac{1}{2}$
2	$\frac{1}{4}$	0	$\frac{1}{2} \cdot \frac{1}{2} + 0 = \frac{1}{4}$
3	$\frac{1}{8}$	0	$\frac{1}{2} \cdot \frac{1}{4} + 0 = \frac{1}{8}$
4	$\frac{1}{16}$	0	$\frac{1}{2} \cdot \frac{1}{8} + 0 = \frac{1}{16}$

$$h[n] = \left[1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots \right]$$

$$= \left(\frac{1}{2} \right)^n u[n]$$

(1/4) δ v. u in final answer
 (-1/2) only 2 terms
 (-1/4) a₁ sign reversal
 (-1/2) step response

(question #3 on back)

3. (6 points) Find the result of convolving $x[n] = \{3, 2, 1\}$ with $h[n] = \{2, -1, 5\}$.

$$y[n] = \sum_k \underbrace{x[k]}_{\substack{\uparrow \\ \text{scale by} \\ \text{sample @} \\ \text{time } k}} \underbrace{h[n-k]}_{\substack{\uparrow \\ \text{shift impulse response} \\ \text{right by } k \text{ samples}}}$$

k	x[k]	n	0	1	2	3	4
0	3		21	-3	15		
1	2			14	-2	10	
2	1				7	-1	5
			<u>21</u>	<u>11</u>	<u>20</u>	<u>9</u>	<u>5</u>

$$y[n] = \{ \underline{21}, 11, 20, 9, 5 \}$$

Check: Area Property: $A_x = \sum_n x[n] = 3+2+1=6$
 $A_h = 2-1+5=11$
 $A_y = 21+11+20+9+5=66$
 $A_y \stackrel{?}{=} A_x \cdot A_h = 6 \cdot 11 = 66 \rightarrow \checkmark$

Width Property
 $W_x = n_{\text{end}} - n_{\text{start}} + 1 = 2 - 0 + 1 = 3 \text{ samples}$
 $W_h = 2 - 0 + 1 = 3$
 $W_y \stackrel{?}{=} 4 - 0 + 1 = 5$
 $W_y \stackrel{?}{=} W_x + W_h - 1 = 3 + 3 - 1 = 5 \rightarrow \checkmark$