Name *Answers*

EE-3221-11 – Dr. Durant – Quiz 5
Winter 2017-'18, Week 5

z-transform: $X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$

1. (2 points) ***Calculate*** the first 4 samples of the unit ***impulse*** response of y(n) = 0.8 y(n–1) + 2 x(n) – 0.6 x(n–1) – 0.8 x(n-2). Recall that the impulse response is y(n) when x(n) = δ(n).

2. (2 points) ***Re-write*** the equation in standard form and then ***indicate*** the name of each coefficient (a_1, etc.).

3. (1 point) Calculate the z-transform of x = [5 0 2 -4 0 3].

4. (3 points) ***Multiply*** the z-transform you just calculated by z^{-3}. Then, take the inverse z-transform and give the ***resulting x(n)***. What can you ***conclude*** about the effect of multiplying the z-transform by z^{-3}?

5. (2 points) ***Find*** the z-transform of x = [1 1/2 1/4 1/8 1/16 ...]. Note that this is a causal geometric sequence with ratio 1/2. For full credit, present your answer in ***closed form*** (not as an infinite sum).

(1)

	2	−0.6	−0.8	0.8	
n	x(n)	x(n−1)	x(n−2)	y(n−1)	y(n)
0	1	0	0	0	2
1	0	1	0	2	1
2	0	0	1	1	0
3	0	0	0	0	0

(! AR systems are not always IIR!)

(2) $y(n) \underbrace{-0.8}_{a_1} y(n-1) = \underbrace{2}_{b_0} x(n) \underbrace{-0.6}_{b_1} x(n-1) \underbrace{-0.8}_{b_2} x(n-2)$

$a_0 = 1$

(3) $X(z) = 5 + 2z^{-2} - 4z^{-3} + 3z^{-5}$

(4) $X^d(z) = 5z^{-3} + 2z^{-5} - 4z^{-6} + 3z^{-8}$

$x^d(n) = [0\ 0\ 0\ 5\ 0\ 2\ -4\ 0\ 3]$

z^{-3} causes a delay of 3 samples

(5) $X(z) = 1 + \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2} + \frac{1}{8}z^{-3} + \ldots = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^{-n} = \frac{1}{1-\frac{1}{2}z^{-1}} = \frac{z}{z-\frac{1}{2}}$

Name Answers

EE-3221-41 – Dr. Durant – Quiz 5
Winter 2017-'18, Week 5

z-transform: $X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$

1. (2 points) ***Calculate*** the first 4 samples of the unit ***step*** response of y(n) = 0.5 y(n–1) + 3 x(n) - x(n–1). Recall that the step response is y(n) when x(n) = u(n).
2. (2 points) ***Re-write*** the equation in standard form and then ***indicate*** the name of each coefficient (a_1, etc.).
3. (1 point) Calculate the z-transform of x = [6 3 0 -4 0 0 2].
4. (3 points) ***Multiply*** the z-transform you just calculated by z^{-2}. Then, take the inverse z-transform and give the ***resulting x(n)***. What can you ***conclude*** about the effect of multiplying the z-transform by z^{-2}?
5. (2 points) ***Find*** the z-transform of x = [1 -1/3 1/9 -1/27 1/81 ...]. Note that this is a causal geometric sequence with ratio -1/3. For full credit, present your answer in ***closed form*** (not as an infinite sum).

(1)

n	x(n) (3)	x(n-1) (-1)	y(n-1) (0.5)	y(n)
0	1	0	0	3
1	1	1	3	$3-1+\frac{3}{2}=7/2$
2	1	1	7/2	$3-1+\frac{7}{2}=11/2$
3	1	1	11/2	$3-1+\frac{11}{2}=15/2$

math errors for n = {2,3}, for n=2, y(2) = 3-1+7/4 = 15/4 = 3 3/4
For n = 3, y(3) = 3-1+15/8 = 31/8 = 3 7/8

(2) $y(n) - \underbrace{\tfrac{1}{2}}_{a_1} y(n-1) = \underset{b_0}{3}\, x(n) \underset{b_1=-1}{-} x(n-1)$, $a_0 = 1$

(3) $X(z) = 6 + 3z^{-1} - 4z^{-3} + 2z^{-6}$

(4) $X_d(z) = 6z^{-2} + 3z^{-3} - 4z^{-5} + 2z^{-8}$

$x_d(n) = [0\ 0\ 6\ 3\ 0\ -4\ 0\ 0\ 2] = x(n-2)$

z^{-2} multiplication in X(z) delays x(n) by 2 samples

(5) $X(z) = 1 - \frac{1}{3}z^{-1} + \frac{1}{9}z^{-2} - \frac{1}{27}z^{-3} + \dots = \sum_{n=0}^{\infty}\left(\frac{-1}{3}\right)^n z^{-n} = \sum_{n=0}^{\infty}\left(\frac{-1}{3}\cdot\frac{1}{z}\right)^n = \frac{1}{1-\frac{-1}{3}z^{-1}} = \frac{z}{z+1/3}$