

EE-3220 - Dr. Durant - Quiz 5  
Winter 2016-'17, Week 6

Given the difference equation  $y(n] = 0.9 y[n-1] + 2 x[n]$

1. (2 points) Take the z-transform of both sides of the equation. Remember,  $z^{-1}$  represents a sample delay.

$$Y(z) = 0.9 z^{-1} Y(z) + 2X(z)$$

2. (2 points) Solve the above equation for the transfer function  $H(z)$ .

$$Y(z)(1 - 0.9z^{-1}) = 2X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{2}{1 - 0.9z^{-1}} = \frac{2z}{z - 0.9}$$

(-1) recip

3. (1 point) Write out the first 5 terms of  $x[n]$  given that  $X(z) = \frac{z}{z+1}$ .

Form:  $\frac{z}{z-a}$ ,  $a = -1$ , geometric series

$$x[n] = \{1, -1, 1, -1, 1\}$$

4. (2 points) Calculate  $Y(z)$  and simplify it using partial fractions.

$$Y(z) = H(z) X(z) = \frac{2z^2}{(z-0.9)(z+1)}$$

$$\frac{Y(z)}{z} = \frac{2z}{(z-0.9)(z+1)} = \frac{A}{z-0.9} + \frac{B}{z+1}$$

$$2z = A(z+1) + B(z-0.9)$$

$$A+B = 2 \quad A - 0.9B = 0$$

$$1.9B = 2 \quad B = \frac{2}{1.9} = \frac{20}{19}$$

$$A = 2 - B = 2 - \frac{20}{19} = \frac{18}{19}$$

$$Y(z) = \frac{18/19 \cdot z}{z-0.9} + \frac{20/19 \cdot z}{z+1}$$

5. (1 point) Calculate the z-transform of  $x = [2 \ 6 \ -5]$ , which starts at  $n = 2$ .

$$X(z) = 2z^{-2} + 6z^{-3} - 5z^{-4}$$

6. (2 points) Calculate the inverse z-transform of  $X(z) = z^{-2} \frac{z}{z-1} - z^{-1} \frac{z}{z+0.2}$

$$x[n] = u[n-2] + (-0.2)^{n-1} u[n-1]$$