Name *Answers*

EE-3220-11 – Dr. Durant – Quiz 5
Winter 2015-'16, Week 5

Given the difference equation y(n) = 0.75 y(n-1) + 0.5 x(n) – 0.25 x(n-1)

1. (2 points) Take the DTFT of both sides of the equation. Recall that the DTFT of a delayed signal, $y(n-k)$, is $e^{-j\omega k}Y(e^{j\omega})$.
2. (2 points) Solve the above equation for transfer function $H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})}$
3. (1 point) Evaluate H at $\omega = \pi/2$.
4. (~~1~~ 2 point) What is the steady state response if the system input is $x_4(n) = \cos(\pi/2\ n)$? Remember, system delays are represented by negative angles in the DTFT H. Your answer should be an equation that allows you to find the steady response at any sample number, n.

(1) $Y(e^{j\omega}) = \frac{3}{4} Y(e^{j\omega}) e^{-j\omega} + \frac{1}{2} X(e^{j\omega}) - \frac{1}{4} X(e^{j\omega}) e^{-j\omega}$

$Y(e^{j\omega})\left(1 - \frac{3}{4}e^{-j\omega}\right) = X(e^{j\omega})\left(\frac{1}{2} - \frac{1}{4}e^{-j\omega}\right)$

(2) $H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{\frac{1}{2} - \frac{1}{4}e^{-j\omega}}{1 - \frac{3}{4}e^{-j\omega}} \cdot \frac{4e^{j\omega}}{4e^{j\omega}} = \frac{2e^{j\omega} - 1}{4e^{j\omega} - 3}$ (-1) recip.

(3) $\omega = \frac{\pi}{2}$ $e^{j\omega} = \cos\left(\frac{\pi}{2}\right) + j\sin\left(\frac{\pi}{2}\right) = 0 + j \cdot 1 = j$

$H(e^{j\omega})\Big|_{\omega=\frac{\pi}{2}} = \frac{2 \cdot j - 1}{4 \cdot j - 3} = \frac{1-2j}{3-4j} \cdot \frac{3+4j}{3+4j} = \frac{11-2j}{25} = 0.44 - 0.08j$ $\left(\frac{1}{2.2+0.4j}\right)$

$= \frac{\sqrt{125}}{25} \angle \tan^{-1}\left(\frac{-2}{11}\right) = \frac{1}{\sqrt{5}} \angle \tan^{-1}\left(\frac{-2}{11}\right) \doteq 0.4472 \angle -0.1799$

(4) H gives us gain & delay:

$y(n) = \frac{1}{\sqrt{5}} \cos\left(\frac{\pi}{2} n + \underbrace{\tan^{-1}\left(\frac{-2}{11}\right)}_{\text{this is negative, so we have a delay}}\right)$

5. (1 point) Now, let ω = 0. What is the value of ej$^{\omega}$?
6. (1 point) What is the value of the DTFT H at ω = 0?
7. (1 point) What sort of input would you give to the system to confirm that your calculated value of H is correct?
8. (Bonus point) Explain, using properties of the DTFT, why H at π must be a real number (its imaginary part must be 0). Recall that $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n}$, $X(e^{j(\omega+2\pi)}) = X(e^{j\omega})$ and, for real signals, $X(e^{-j\omega}) = X^{*}(e^{j\omega})$.

(5) $e^{j0} = e^{0} = 1$

(6) $H(e^{j\omega}) = \frac{2e^{j\omega}-1}{4e^{j\omega}-3} = \frac{2-1}{4-3} = \frac{1}{1} = 1$

(7) DC input.
Output DC voltage should equal input voltage since gain at $\omega = 0$ is 1.

(8) $X(\pi) = X(-\pi)$ applying 2π period
$X(\pi) = X^{*}(-\pi)$ symmetry for $x(n) \in \boxed{\mathbb{R}^N}$ real sequence
$X(-\pi) = X^{*}(-\pi)$ combine 2 above equation
$X(-\pi) \in \mathbb{R}$ self-conjugate ∴ real
$X(\pi) \in \mathbb{R}$ substitute using first equation