Name *answers*

25

EE-3220-11 – Dr. Durant – Quiz 5
Winter 2014-'15, Week 5

Given the difference equation y(n) = -0.4 y(n-1) + 0.9 x(n)

1. (2 points) Take the z-transform of both sides of the equation. Remember, z^{-1} represents a sample delay.

$$Y(z) = -0.4z^{-1}Y(z) + 0.9X(z)$$

2. (2 points) Solve the above equation for the transfer function H(z).

$$Y(z)(1+0.4z^{-1}) = 0.9X(z)$$
$$H(z) = \frac{Y(z)}{X(z)} = \frac{0.9}{1+0.4z^{-1}} = \frac{0.9z}{z+0.4}$$

3. (2 points) Let the input x(n) be the causal sequence [1 -½ ¼ -1/8 1/16 ...]. Note that this is a geometric series with ratio -1/2. Calculate X(z).

$$\boxed{X(z) = \frac{z}{z+0.5}}$$

Proof: $X(z) = \sum_{n=0}^{\infty} \left(-\frac{1}{2}\right)^n z^{-n} = \sum_{n=0}^{\infty} \left(\frac{-1}{2z}\right)^n = \frac{1}{1-\frac{-1}{2z}} = \frac{z}{z+\frac{1}{2}}$

ROC: $\left|\frac{1}{2z}\right| < 1$

$|z| > \frac{1}{2}$

4. (1 point) Calculate Y(z) based on H(z) and X(z) above. You DO NOT need to simplify it using partial fractions.

$Y(z) = H(z)X(z) = \boxed{\frac{0.9z}{z+0.4} \cdot \frac{z}{z+0.5}}$ ← answer

More: $Y(z)/z = \frac{0.9z}{(z+.4)(z+.5)} = \frac{A}{z+.4} + \frac{B}{z+.5} \rightarrow 0.9z = A(z+.5) + B(z+.4) \rightarrow \begin{matrix} A+B=0.9 \\ .5A+.4B=0 \end{matrix} \rightarrow -\frac{4}{5}B + B = 0.9$

$B = 4.5$

$A = -3.6$

$Y(z) = \frac{-3.6z}{z+.4} + \frac{4.5z}{z+.5} \rightarrow y(n) = [-3.6(-0.4)^n + 4.5(-0.5)^n]\, u(n)$

5. (1 point) Calculate the z-transform of x = [-7 5 2], which starts at ***n=2***.

n 2 3 4

$$X(z) = -7z^{-2} + 5z^{-3} + 2z^{-4}$$

6. (2 points) Calculate the inverse z-transform of $X(z) = \left(\frac{z}{z-1} - z^{-2}\frac{z}{z-1}\right)$.

another way: $\frac{z}{z-1}\left(1 - z^{-2}\right) = \frac{1}{z-1}\left(z - z^{-1}\right) = \frac{z-\frac{1}{z}}{z-1} = \frac{(z^2-1)/z}{z-1} = \frac{z+1}{z} = 1 + z^{-1}$

$x(n) = u(n) - u(n-2) = [1\ 1]$

Name *Answers*

26

EE-3220-21 – Dr. Durant – Quiz 5
Winter 2014-'15, Week 5

Given the difference equation y(n) = 0.8 y(n-1) + 0.3 x(n)

1. (2 points) Take the z-transform of both sides of the equation. Remember, z^{-1} represents a sample delay.

$$Y(z) = 0.8z^{-1}Y(z) + 0.3X(z)$$

2. (2 points) Solve the above equation for the transfer function H(z).

$$Y(z)(1-0.8z^{-1}) = 0.3X(z)$$
$$H(z) = \frac{Y(z)}{X(z)} = \frac{0.3}{1-0.8z^{-1}} = \frac{0.3z}{z-0.8}$$

3. (2 points) Let the input x(n) be the causal sequence [1 -1 1 -1 1 -1 ...]. Note that this is a geometric series with ratio -1. Calculate X(z).

$$X(z) = \frac{z}{z - -1} = \frac{z}{z+1}$$

4. (1 point) Calculate Y(z) based on H(z) and X(z) above. You DO NOT need to simplify it using partial fractions.

$$Y(z) = H(z)X(z) = \frac{z}{z+1} \cdot \frac{0.3z}{z-0.8} = \frac{0.3z^2}{(z+1)(z-0.8)}$$

5. (1 point) Calculate the z-transform of x = [-4 5 3 2], which starts at ***n=-1***.

$$X(z) = -4z + 5 + 3z^{-1} + 2z^{-2}$$

6. (2 points) Calculate the inverse z-transform of $X(z) = \frac{z}{z-0.5} - z^{-3}\frac{z}{z+0.3}$

$$x(n) = 0.5^n u(n) - (-0.3)^{n-3} u(n-3)$$