Name Answers

EE-3220-21 – Dr. Durant – Quiz 5
Winter 2013-'14, Week 6

Given the difference equation y(n) = -0.5 y(n-1) + 0.5 x(n)

1. (1 point) Take the z-transform of both sides of the equation. Remember, z^{-1} represents a sample delay.

   $Y(z) = -0.5 z^{-1} Y(z) + 0.5 X(z)$

2. (2 points) Solve the above equation for the transfer function H(z).

   $Y(z)(1+0.5z^{-1}) = 0.5X(z)$

   $H(z) = \frac{Y(z)}{X(z)} = \frac{0.5}{1+0.5z^{-1}} = \frac{0.5z}{z+0.5}$

3. (2 points) Let the input x(n) be the causal sequence [1 ½ ¼ 1/8 1/16 ...]. Note that this is a geometric series with ratio +1/2. Calculate X(z).

   $X(z) = \frac{z}{z-a} = \frac{z}{z-0.5}$

4. (1 point) Calculate Y(z) based on H(z) and X(z) above. You DO NOT need to simplify it using partial fractions.

   $Y(z) = H(z)X(z) = \frac{0.5z^2}{(z+0.5)(z-0.5)}$

5. (1 point) What are the roots of your denominator polynomial for Y(z)?

   $p = \{-0.5, 0.5\}$

6. (2 points) ~~Which root corresponds to the~~ ~~*steady state*~~ ~~response?~~ Explain ~~why~~. how these roots correspond to y(n) decaying toward ∅.

   ~~0.5: it is the part due to the system~~
   Each pole (denominator root) has magnitude < 1, indicating a decay for a causal system.

7. (1 point) Calculate the z-transform of x = [5 2 -7], which starts at *n=2*.

   $X(z) = \sum_{n=2}^{4} x(n) z^{-n} = 5z^{-2} + 2z^{-3} - 7z^{-4}$

Name Answers

EE-3220-41 – Dr. Durant – Quiz 5
Winter 2013-'14, Week 6

Given the difference equation y(n) = 0.5 y(n-1) + 0.5 x(n)

1. (1 point) Take the z-transform of both sides of the equation. Remember, z^{-1} represents a sample delay.

$$Y(z) = 0.5 z^{-1} Y(z) + 0.5\ X(z)$$

2. (2 points) Solve the above equation for the transfer function H(z).

$$Y(z)\left(1 - 0.5 z^{-1}\right) = 0.5 X(z)$$
$$H(z) = \frac{Y(z)}{X(z)} = \frac{0.5}{1 - 0.5 z^{-1}} = \frac{0.5 z}{z - 0.5}$$

3. (2 points) Let the input x(n) be the causal sequence [1 -1 1 -1 1 -1 ...]. Note that this is a geometric series with ratio -1. Calculate X(z).

$$X(z) = \frac{z}{z-a} = \frac{z}{z+1}$$

4. (1 point) Calculate Y(z) based on H(z) and X(z) above. You DO NOT need to simplify it using partial fractions.

$$Y(z) = H(z)X(z) = \frac{0.5 z^2}{(z+1)(z-0.5)}$$

5. (1 point) What are the roots of your denominator polynomial for Y(z)?

$$p = \{-1,\ 0.5\}$$

6. (2 points) Which root corresponds to the *transient* response? Explain why.

0.5: it is the decaying part due to the system.

acceptable answer since input does not decay

key part of answer

7. (1 point) Calculate the z-transform of x = [3 2 -4 5], which starts at n=0.

$$X(z) = \sum_{n=0}^{3} x(n) z^{-n} = 3 + 2z^{-1} + -4z^{-2} + 5z^{-3}$$