Name *Answers*

EE-3220-11 – Dr. Durant – Quiz 5
Spring 2015, Week 5

Given the difference equation y(n) = 0.5 y(n-1) + 5 x(n) – 2 x(n-1)

1. (2 points) Take the z-transform of both sides of the equation. Remember, z^{-1} represents a sample delay.

$$Y(z) = \frac{1}{2} z^{-1} Y(z) + 5X(z) - 2z^{-1}X(z)$$

2. (2 points) Solve the above equation for the transfer function H(z).

$$Y(z)\left(1-\frac{1}{2}z^{-1}\right) = X(z)\left(5-2z^{-1}\right)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{5-2z^{-1}}{1-\frac{1}{2}z^{-1}} = \frac{5z-2}{z-\frac{1}{2}} = \frac{10z-4}{2z-1}$$

preferred form

3. (2 points) Let the input x(n) be the causal sequence [1 ½ ¼ 1/8 1/16 ...]. Note that this is a geometric series with ratio +1/2. Calculate X(z).

$$X(z) = \frac{z}{z-1/2} = \frac{1}{1-\boxed{\frac{1}{2}z^{-1}}}$$

initial term
ratio in geometric series
preferred form

4. (1 point) Calculate Y(z) based on H(z) and X(z) above. You DO NOT need to simplify it using partial fractions.

$$Y(z) = H(z)X(z) = \frac{5z-2}{z-1/2} \cdot \frac{z}{z-1/2} = \frac{5z^2-2z}{(z-1/2)^2}$$

5. (1 point) Calculate the z-transform of x = [6 -5 2], which starts at ***n=-2***.
↑

$$X(z) = 6z^2 - 5z + 2$$

delay by 2
$(0.1)^n u(n)$

6. (2 points) Calculate the inverse z-transform of $X(z) = \frac{z}{z-1} - z^{-2}\frac{z}{z-0.1}$

$$x(z) = u(n) - 0.1^{n-2}u(n-2)$$