Name *Answers*

EE-3220 – Dr. Durant – Quiz 4
Winter 2016-'17, Week 5

Reminder: The DTFT is defined by $X(\omega) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n}$. (Sometimes the DTFT is symbolized by $X(e^{j\omega})$ as a shorthand notation emphasizing that the function X is often generalized to be defined anywhere in the complex plane, $X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n}$. In this case the DTFT is found by evaluating X at points on the unit circle, $1\angle\omega$, that is, letting $z = e^{-j\omega}$.)

1. (1 point) Let f_s = 10000 Hz, f_1 = 3000 Hz, and f_2 = 8000 Hz. ***Calculate*** the digital frequencies, ω_n, for f_1 and f_2. Recall that the digital frequency is how many radians a sinusoid moves through between samples. For example, if a signal is sampled 10 times per period, its digital frequency is 2π/10. Do ***not*** make any adjustments for aliasing.

2. (1 point) ***Explain*** whether any of the 2 frequencies above are aliased. For each frequency that is aliased, assuming it was not stopped by a suitable antialias filter, ***calculate*** what frequency in hertz would be observed at the output of the system due to aliasing.

3. (1 point) Let $x(n) = \cos((\pi/3)\, n)\, (u(n+2) - u(n-3))$. Calculate the samples of $x(n)$. Recall that $\cos(\pi/3) = \cos(60°) = ½$.

4. (1 point) Apply the DTFT definition to calculate $X(e^{j\omega})$ based on your answer to the previous question.

(1) $\omega_1 = \frac{F_1}{f_s} 2\pi = \frac{3000}{10000} 2\pi = \frac{3\pi}{5}$

$\omega_2 = \frac{F_2}{f_s} 2\pi = \frac{8000}{10000} 2\pi = \frac{8\pi}{5}$

(2) $|\omega_2| > \pi$ ∴ it aliases. The alias is at $\frac{8\pi}{5} - 2\pi = \frac{-2\pi}{5}$

In hertz: $8000 - 10000 = \boxed{-2000\text{ Hz}}$

or, 2000 Hz w/ an inverted ~~sinusoid~~ sine (odd) portion.

(3) $u(\cdot) - u(\cdot) \Rightarrow -2 \le n \le +2$ (region where $u - u = 1$ (not 0))

n	$\cos(\frac{\pi}{3} n)$
-2	-1/2
-1	1/2
0	1
1	1/2
2	-1/2

(4) $X(\omega) = -\frac{1}{2}e^{j2\omega} + \frac{1}{2}e^{j\omega} + 1 + \frac{1}{2}e^{-j\omega} - \frac{1}{2}e^{-j2\omega}$

Given the difference equation y(n) = 0.5 y(n-1) + 0.75 x(n) – 0.25 x(n-1)

5. (2 points) Take the DTFT of both sides of the equation. Recall that the DTFT of a delayed signal, $y(n-k)$, is $e^{-j\omega k}Y(e^{j\omega})$.
6. (2 points) Solve the above equation for transfer function $H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})}$
7. (1 point) Evaluate H at $\omega = \pi/2$.
8. (1 point) Based on your value for H, what would be the gain and phase delay in radians for an input sinusoid having frequency of ω radians/sample?

(5) $Y(\omega) = 0.5\,Y(\omega)e^{-j\omega} + 0.75\,X(\omega) - 0.25\,X(\omega)e^{-j\omega}$

$Y(\omega)(1 - 0.5e^{-j\omega}) = X(\omega)(0.75 - 0.25e^{-j\omega})$

(6) $H = \frac{Y}{X} = \underset{\text{OK}}{\frac{0.75 - 0.25e^{-j\omega}}{1 - 0.5e^{-j\omega}}} \cdot \frac{4}{4} = \underset{\text{better}}{\frac{3 - e^{-j\omega}}{4 - 2e^{-j\omega}}} \cdot \frac{e^{j\omega}}{e^{j\omega}} = \underset{\text{best}}{\frac{3e^{j\omega} - 1}{4e^{j\omega} - 2}}$

(7) $H\big|_{\omega = \frac{\pi}{2}} = \frac{3j - 1}{4j - 2} \cdot \frac{4j + 2}{4j + 2} = \frac{-12 + 2j - 2}{-16 - 4} = \frac{-14 + 2j}{-20} = \frac{7 - j}{10} = \frac{1}{\sqrt{2}} \angle \overbrace{-0.1419}^{\tan^{-1}\left(\frac{-1}{7}\right)}$

(8) Gain $= |H| = \frac{1}{\sqrt{2}} = 0.707$

ϕ delay $= \angle H = -0.1419$