Name *Answers*

# EE-3220-11 – Dr. Durant – Quiz 4
## Winter 2015-'16, Week 4

Reminder: The DTFT is defined by $X(\omega) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n}$. (Sometimes the DTFT is symbolized by $X(e^{j\omega})$ as a shorthand notation emphasizing that the function $X$ is often generalized to be defined anywhere in the complex plane, $X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n}$. In this case the DTFT is found by evaluating $X$ at points on the unit circle, $1\angle\omega$, that is, letting $z = e^{-j\omega}$.)

1. (2 points) Let $f_s$ = 1000 Hz, $f_1$ = 0 Hz, $f_2$ = 200 Hz, and $f_3$ = 700 Hz. ***Calculate*** the digital frequencies, $\omega_n$, for each frequency, $f_n$. for $f_1$ through $f_3$. Recall that the digital frequency is how many radians a sinusoid moves through between samples. For example, if a signal is sampled 10 times per period, its digital frequency is 2π/10. Do ***not*** make any adjustments for aliasing.

2. (2 points) ***Explain*** whether any of the 3 sinusoids above are aliased. For each frequency that is aliased, assuming it was not stopped by a suitable antialias filter, ***calculate*** what frequency in hertz would be observed at the output of the system due to aliasing.

(1) $w = \frac{F}{F_S} 2\pi$

$w_1 = \frac{0}{1000} 2\pi = \boxed{0}$

$w_2 = \frac{200}{1000} 2\pi = \boxed{\frac{2\pi}{5}}$

$w_3 = \frac{700}{1000} 2\pi = \boxed{\frac{7\pi}{5}}$

(2) Only $w_3$ is aliased.

Apply $2\pi$ periodicity

$w_3^* = w_3 - 2\pi = \frac{-3\pi}{5}$

$|w_3^*| < |w_3|$ ∴ the lower frequency $w_3^*$ is the aliased frequency that will be reconstructed from samples of a sinusoid at $w_3 = \frac{7\pi}{5}$

In hertz: $-\frac{3\pi}{5} = \frac{F}{1000\,Hz} 2\pi$

$F = -300\,Hz$

$\boxed{300\,Hz}$

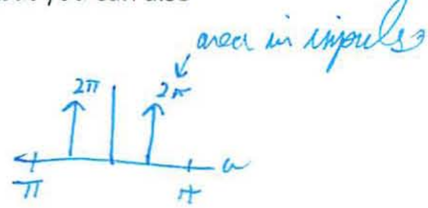
$-\frac{1}{2}$ shift π, but o.w. correct, just as

$-\frac{3}{4}$ alias correctly id'd, but no adj. freq.

3. (1 point) Let $x_1(n) = 2\cos((\pi/2)\, n)$. Calculate $X_1(e^{j\omega})$. Recall that the DTFT of $\cos(\omega_0 n)$ is $\pi(\delta(\omega-\omega_0)+\delta(\omega+\omega_0))$.
4. (1 point) Let $x_2(n) = x_1(n)\,(u(n+2) - u(n-2))$. Calculate the samples of $x_2(n)$.
5. (1 point) Calculate $X_2(e^{j\omega})$ based on your answer to the previous question. Note: your answer will look a lot different than the other DTFT you calculated.
6. (1 point) Explain the following property of the DTFT: $X\left(e^{j(\omega+2\pi)}\right) = X\left(e^{j\omega}\right)$.
7. (2 points) How is the value of the DTFT at $-\omega$ related to its value at $\omega$? Assume x(n) is a real signal. (You may just state the answer if you remember it from the book, but you can also derive it from the DTFT definition; the first step is evaluating $X(\omega)$ at $-\omega$.

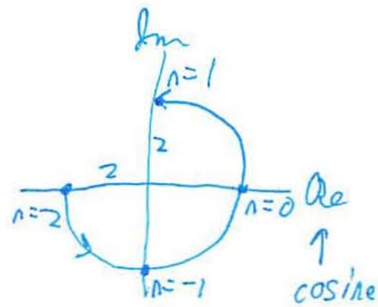
$A = 2 \quad \omega = \frac{\pi}{2}$

(3) $X_1(e^{j\omega}) = 2\pi\left(\delta\left(\omega - \frac{\pi}{2}\right) + \delta\left(\omega + \frac{\pi}{2}\right)\right)$




(4) $u(n+2) - u(n-2) \Rightarrow -2 \le n < 2$

| $n$ | $2\cos\left(\frac{\pi}{2}n\right)$ |
|---|---|
| $-2$ | $2\cos(-\pi) = -2$ |
| $-1$ | $2\cos\left(-\frac{\pi}{2}\right) = 0$ |
| $0$ | $2\cos(0) = 2$ |
| $1$ | $2\cos\left(\frac{\pi}{2}\right) = 0$ |




$$\boxed{x_2(n) = \{-2 \;\; 0 \;\; \underset{\uparrow}{2}\}}$$

(5) $$X_2(e^{j\omega}) = \sum_n x(n) e^{-j\omega n} = \overbrace{-2e^{j2\omega}}^{n=-2} + \overbrace{2e^{j0\omega}}^{n=0}$$
$$= \boxed{2 - 2e^{j2\omega}}$$

(6) The DTFT is periodic with period $2\pi$. That is, a sinusoid moving $2k\pi$, $k \in \mathbb{Z}$ faster will generate the same samples, so its DTFT value must be the same.

integer

(7) $X(-\omega) = \sum_n x(n) \underbrace{e^{+j\omega n}}_{\text{conjugate of DTFT exponential}}$ $\therefore$ $\boxed{X(-\omega) = X^*(\omega)}$ since $x(n) \in \mathbb{R}$ does not change the angle, just the magnitude of each term

conjugate

Sum of conjugates = conjugate of sum