Name _***ANSWERS***_

EE-3220-11 – Dr. Durant – Quiz 4
Winter 2014-'15, Week 4

1. (1 point) Let $x_1(n) = \cos((\pi/2)\,n) + 0.5\cos((\pi/4)\,n)$. Calculate $X_1(e^{j\omega})$. Recall that the DTFT of $\cos(\omega_0 n)$ is $\pi(\delta(\omega-\omega_0)+\delta(\omega+\omega_0))$.

$X_1(e^{j\omega}) = F\{\cos((\pi/2)\,n) + 0.5\cos((\pi/4)\,n)\} = F\{\cos((\pi/2)\,n)\} + 0.5\,F\{\cos((\pi/4)\,n)\}$

$= \pi\,(\delta(\omega-\pi/2)+\delta(\omega+\pi/2)) + 0.5\,\pi\,(\delta(\omega-\pi/4)+\delta(\omega+\pi/4))$

2. (1 point) Let $x_2(n) = x_1(n)\,(u(n) - u(n-4))$. Calculate the samples of $x_2(n)$.

n	0	1	2	3
cos((π/2) n)	1	0	-1	0
0.5 cos((π/4) n)	½	1/(2√2)	0	-1/(2√2)
Σ	1 ½	1/(2√2)	-1	-1/(2√2)

3. (1 point) Calculate $X_2(e^{j\omega})$ based on your answer to the previous question. Recall that $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n}$. Note: your answer will look a lot different than your answer to the first question.

$$X(e^{j\omega}) = 1\tfrac{1}{2} + \frac{1}{2\sqrt{2}}e^{-j\omega n} - e^{-j2\omega} - \frac{1}{2\sqrt{2}}e^{-j3\omega}$$

Given the difference equation y(n) = 0.5 y(n-1) - 0.5 x(n)

4. (2 points) Take the DTFT of both sides of the equation. Recall that the DTFT of a delayed signal, $y(n-k)$, is $e^{-j\omega k}Y(e^{j\omega})$.

$Y(e^{j\omega}) = 0.5e^{-j\omega}Y(e^{j\omega}) - 0.5\,X(e^{j\omega})$

5. (2 points) Solve the above equation for transfer function $H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})}$

$Y(e^{j\omega}) - 0.5e^{-j\omega}Y(e^{j\omega}) = -0.5\,X(e^{j\omega})$

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{-0.5}{1-0.5e^{-j\omega}} = \frac{-1}{2-e^{-j\omega}}$$

6. (1 point) Let f_s = 800 Hz, f_1 = 200 Hz, and f_2 = 400 Hz. Calculate the digital frequencies, ω_n, for each frequency, f_n. for f_1 through f_2. Recall that the digital frequency is how many radians a sinusoid moves through between samples. For example, if a signal is sampled 10 times per period, its digital frequency is 2π/10.

$\omega_1 = f/fs\ (2\pi) = 200/800\ (2\pi) = \pi/2$

$\omega_2 = f/fs\ (2\pi) = 400/800\ (2\pi) = \pi$

7. (2 points) Evaluate *H* at the digital frequencies calculated above.

$$\omega_1 : \frac{-1}{2-e^{-j\omega}} = \frac{-1}{2-e^{-j\pi/2}} = \frac{-1}{2--j} = \frac{-1}{2+j} = \frac{-2+j}{5}$$

$$\omega_2 : \frac{-1}{2-e^{-j\omega}} = \frac{-1}{2-e^{-j\pi}} = \frac{-1}{2--1} = \frac{-1}{3}$$

8. (1 point) What do these values of H tell you about the steady state response to sinusoids?

They give the magnitude and phase shift of sinusoids at the corresponding frequencies. For example, a 400 Hz wave will be scaled down by a factor of 3 and inverted (or phase shifted by 180 degrees). They do not tell us about the transient response when the sinusoid is first applied.

Name _***ANSWERS***_

EE-3220-12 – Dr. Durant – Quiz 4
Winter 2014-'15, Week 4

1. (1 point) Let $x_1(n) = \cos((\pi/2)\, n) + 2\cos((\pi/3)\, n)$. Calculate $X_1(e^{j\omega})$. Recall that the DTFT of $\cos(\omega_0 n)$ is $\pi(\delta(\omega-\omega_0)+\delta(\omega+\omega_0))$.

$X_1(e^{j\omega}) = F\{\cos((\pi/2)\, n) + 2\cos((\pi/3)\, n)\} = F\{\cos((\pi/2)\, n)\} + 2F\{\cos((\pi/3)\, n)\}$

$= \pi\,(\delta(\omega-\pi/2)+\delta(\omega+\pi/2)) + 2\pi\,(\delta(\omega-\pi/3)+\delta(\omega+\pi/3))$

2. (1 point) Let $x_2(n) = x_1(n)\,(u(n) - u(n-4))$. Calculate the samples of $x_2(n)$.

n	0	1	2	3
cos((π/2) n)	1	0	-1	0
2 cos((π/3) n)	2	1	-1	-2
Σ	3	1	-2	-2

3. (1 point) Calculate $X_2(e^{j\omega})$ based on your answer to the previous question. Recall that $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n}$. Note: your answer will look a lot different than your answer to the first question.

$$X(e^{j\omega}) = 3 + 1e^{-j\omega n} - 2e^{-j2\omega} - 2e^{-j3\omega}$$

Given the difference equation y(n) = -0.5 y(n-1) + 0.2 x(n)

4. (2 points) Take the DTFT of both sides of the equation. Recall that the DTFT of a delayed signal, $y(n-k)$, is $e^{-j\omega k}Y(e^{j\omega})$.

$Y(e^{j\omega}) = -0.5e^{-j\omega}Y(e^{j\omega}) + 0.2\,X(e^{j\omega})$

5. (2 points) Solve the above equation for transfer function $H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})}$

   $Y(e^{j\omega}) + 0.5e^{-j\omega}Y(e^{j\omega}) = 0.2\,X(e^{j\omega})$

   $$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{0.2}{1 + 0.5e^{-j\omega}} = \frac{2}{10 + 5e^{-j\omega}}$$

6. (1 point) Let f_s = 1000 Hz, f_1 = 250 Hz, and f_2 = 375 Hz. Calculate the digital frequencies ω_1 and ω_2 for f_1 and f_2. Recall that the digital frequency is how many radians a sinusoid moves through between samples. For example, if a signal is sampled 10 times per period, its digital frequency is 2π/10.

$\omega_1 = f/fs\,(2\pi) = 250/1000\,(2\pi) = \pi/2$

$\omega_2 = f/fs\,(2\pi) = 375/1000\,(2\pi) = 3\pi/4$

7. (2 points) Evaluate H at just ω_1.

$$\frac{2}{10 + 5e^{-j\omega}} = \frac{2}{10 + 5e^{-j\pi/2}} = \frac{2}{10 + 5(-j)} = \frac{2(10 + 5j)}{125} = \frac{4 + 2j}{25}$$

8. (1 point) What does this value of H tell you about the steady state response to a 250 Hz sinusoid?

It gives the magnitude and phase shift of sinusoids at the corresponding frequency. For example, a 250 Hz sinusoidal input will result in a 250 Hz sinusoidal output with its magnitude scaled by |H| and its phase advanced by atan(2/4). It does not tell us about the transient response when the sinusoid is first applied.