Name Answers

EE-3220-11 – Dr. Durant – Quiz 4
Spring 2015, Week 4

1. (1 point) Let $x_1(n) = 0.5\cos((\pi/2)\,n) + \cos(\pi\,n)$. Calculate $X_1(e^{j\omega})$. Recall that the DTFT of $\cos(\omega_0 n)$ is $\pi(\delta(\omega-\omega_0)+\delta(\omega+\omega_0))$.

applying linearity:

$$X_1(e^{j\omega}) = \pi\left(0.5\left(\delta\left(\omega-\tfrac{\pi}{2}\right)+\delta\left(\omega+\tfrac{\pi}{2}\right)\right)+\left(\delta(\omega-\pi)+\delta(\omega+\pi)\right)\right)$$

2. (1 point) Let $x_2(n) = x_1(n)\,(u(n+2) - u(n-2))$. Calculate the samples of $x_2(n)$.

n	-3	-2	-1	0	1	2
$u(n+2)$	0	1	1	1	1	1
$u(n-2)$	0	0	0	0	0	1
$-$	0	1	1	1	1	0

Effectively, $n = -2:1$

n	-2	-1	0	1
$\frac{\pi}{2}\cdot n$	$-\pi$	$-\pi/2$	0	$\pi/2$
$0.5\cos(\cdot)$	-0.5	0	0.5	0
πn	-2π	$-\pi$	0	π
$\cos(\cdot)$	1	-1	1	-1
Σ	$\boxed{0.5}$	$\boxed{-1}$	$\boxed{1.5}$	$\boxed{-1}$

↑

3. (1 point) Calculate $X_2(e^{j\omega})$ based on your answer to the previous question. Recall that $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n}$. Note: your answer will look a lot different than your answer to the first question since the sinusoids are truncated in time.

$$X(e^{j\omega}) = 0.5e^{+j\omega 2} - e^{+j\omega} + 1.5 - e^{-j\omega}$$

Given the difference equation y(n) = 0.8 y(n-1) - 0.6 x(n)

4. (2 points) Take the DTFT of both sides of the equation. Recall that the DTFT of a delayed signal, $y(n-k)$, is $e^{-j\omega k}Y(e^{j\omega})$.

$$Y(e^{j\omega}) = 0.8\, e^{-j\omega}\, Y(e^{j\omega}) - 0.6\, X(e^{j\omega})$$

5. (2 points) Solve the above equation for transfer function $H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})}$

$$Y(e^{j\omega})(1 - 0.8e^{-j\omega}) = -0.6\, X(e^{j\omega})$$

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{-0.6}{1 - 0.8e^{-j\omega}}$$

6. (1 point) Let f_s = 1200 Hz, f_1 = 300 Hz, and f_2 = 600 Hz. Calculate the digital frequencies, ω_n, for each frequency, f_n. for f_1 through f_2. Recall that the digital frequency is how many radians a sinusoid moves through between samples. For example, if a signal is sampled 10 times per period, its digital frequency is 2π/10.

$$\omega_1 = \frac{F_1}{fs} 2\pi = \frac{1}{4} 2\pi = \pi/2$$

$$\omega_2 = \frac{F_2}{fs} 2\pi = \frac{1}{2} 2\pi = \pi$$

7. (2 points) Evaluate H at ~~the digital frequencies calculated above~~ your value of ω_1.

$$H(e^{j\omega_1}) = \frac{-0.6}{1 - 0.8e^{-j\pi/2}} = \frac{-0.6}{1 - 0.8 \cdot -j} = \frac{-0.6}{1 + j0.8} \cdot \frac{1 - j0.8}{1 - j0.8} = \frac{j0.48 - 0.6}{1 + 0.64}$$

$$= \frac{j0.48 - 0.6}{1.64} = -0.365\ldots + j0.292\ldots = \frac{-15}{41} + j\frac{12}{41}$$

$$= 0.468\ldots \angle 2.466\ldots$$

$$= 0.468\ldots \angle (0.785\ldots\pi)$$ ← most useful form for angle

$$= 0.468\ldots \angle 141.3^\circ$$

8. (1 point) What ~~do~~ es ~~these~~ is values of H tell you about the steady state response to sinusoids?

The magnitude & angle of H are the gain & phase shift of a sinusoid at that frequency