Name *Answers*

EE-3221-11 – Dr. Durant – Quiz 3
Winter 2017-'18, Week 3

1. Let x(n) = 5 sin(0.72π n). Prove that this signal is periodic.
2. In the expression above, recall that 0.72π is the digital frequency in radians/sample. Explain whether aliasing occurs, that is, whether this frequency can be reconstructed without being confused with a lower frequency.
3. If you were to implement this signal with a look-up table as in lab 2, what is the minimum length of the table (that is, find the fundamental period in samples).
4. Calculate the power of x(n). It is not necessary to calculate values of samples of x(n); you may use whatever shortcut methods you wish, but show your work.
5. Now, let $x_2(n) = 5e^{j0.72\pi n}$. How is this signal related to the original x(n)?
6. What is the power of $x_2(n)$? Comment on how it relates to the power of x(n).

(1) $0.72\pi \overset{?}{=} 2\pi \frac{k}{N}$

$\frac{72}{100} = \frac{18}{25} = \frac{2k}{N}$

$\frac{k}{N} = \frac{9}{25}$

Integer solutions

$N_0 = 25$

∴ periodic

(2) Nyquist: $\omega_{max} < \pi$

$0.72\pi < \pi$

∴ no aliasing

(3) 25 (N_0 from #1)

(4) $P_x = A^2/2 = 25/2 = \boxed{12\frac{1}{2}}$

Or, 1 period: $P_x = \frac{1}{25}\sum_{n=0}^{24} 5^2 \sin^2(0.72\pi n) = \sum_{n=0}^{24} \frac{1}{2} - \underbrace{\cancel{\frac{1}{2}\cos(1.44\pi n)}}_{0} = 25 \cdot \frac{1}{2} = \frac{25}{2}$

$k = 18, N = 25$ ∴ $\sum \cos = 0$

(5) $x(n) = \operatorname{Im}\{x_2(n)\}$

(6) $P_{x_2} = \frac{1}{25}\sum_{n=0}^{24} 5e^{j0.72n}\, 5e^{-j0.72n} = \sum_{n=0}^{24} 1 = 25 = A^2 = \underline{\underline{2P_x}}$

Twice the power: Euler gives sin + cos, same freq., orthogonal signals, ∴ their power <u>adds</u>.

Name Answers

EE-3221-41 – Dr. Durant – Quiz 3
Winter 2017-‘18, Week 3

1. Let x(n) = 3 sin((3π/17) n). Prove that this signal is periodic.

2. In the expression above, recall that 3π/17 is the digital frequency in radians/sample. Explain whether aliasing occurs, that is, whether this frequency can be reconstructed without being confused with a lower frequency.

3. If you were to implement this signal with a look-up table as in lab 2, what is the minimum length of the table (that is, find the fundamental period in samples).

4. Calculate the power of x(n). It is not necessary to calculate values of samples of x(n); you may use whatever shortcut methods you wish, but show your work.

5. Now, let $x_2(n) = 5e^{j(3\pi/17)n}$. How is this signal related to the original x(n)?

6. What is the power of $x_2(n)$? Comment on how it relates to the power of x(n).

(1) Periodic if $\omega = 2\pi\frac{k}{N} \stackrel{?}{=} 3\pi/17$

$\frac{k}{N} = \frac{3}{34}$

Yes. A solution in integers exists
$N_0 = 34$, $k = 3$

Extra details
$x(n) = x(n+N)$
$3\sin\left(\frac{3\pi}{17}n\right) = 3\sin\left(\frac{3\pi}{17}(n+N)\right)$
ϕ difference:
$\frac{3\pi}{17}((n+N)-n) = \frac{3\pi}{17}\cdot N = \frac{3\pi}{17}\cdot 34 = 6\pi = \underbrace{3}_{k}\times 2\pi$
∴ goes through 3 cycles of analog waveform in 34 samples.

(2) Nyquist: $\omega_{max} \stackrel{?}{<} \pi$

$\frac{3\pi}{17} < \pi$

True, no aliasing

(3) 34 (N_0 from #1)

(4) $P_x = A^2/2 = 9/2 = 4\frac{1}{2}$

Or: $P_x = \frac{1}{34}\sum_{n=0}^{33} 3^2 \sin^2\left(\frac{3\pi}{17}n\right) = \frac{9}{34}\sum_{n=0}^{33} \frac{1}{2} - \cancel{\frac{1}{2}\cos\left(\frac{6\pi}{17}n\right)}$
$= \frac{9}{34}\cdot 34\cdot\frac{1}{2} = \frac{9}{2} = 4\frac{1}{2}$

0 since $N_i = \frac{17}{17}$... 17 & 34 is a period
Σ cos over full cycles = 0

(5) $x(n) = \mathrm{Im}\{x_2(n)\}$

(6) $P_{x_2} = \frac{1}{34}\sum_{n=0}^{33} 3^2 e^{j\frac{3\pi}{17}n} e^{-j\frac{3\pi}{17}n} = \frac{9}{34}\sum_{n=0}^{33} 1 = 9 = \underline{\underline{2}}\, P_x$

Twice the power. Equal pwr. in sin & cos. (orthogonal even though w same)