

EE-3220-11 - Dr. Durant - Quiz 3  
Winter 2014-'15, Week 3

1. (2 points) Let  $f_s = 1000$  Hz,  $f_1 = 0$  Hz,  $f_2 = 300$  Hz, and  $f_3 = 600$  Hz. Calculate the digital frequencies,  $\omega_n$ , for each frequency,  $f_n$ , for  $f_1$  through  $f_3$ . Recall that the digital frequency is how many radians a sinusoid moves through between samples. For example, if a signal is sampled 10 times per period, its digital frequency is  $2\pi/10$ . Do **not** make any adjustments for aliasing.

$$\omega_1 = \frac{f_1}{f_s} \cdot 2\pi = 0$$

$$\omega_2 = \frac{f_2}{f_s} \cdot 2\pi = \frac{300}{1000} \cdot 2\pi = \frac{3\pi}{5} = 0.6\pi$$

$$\omega_3 = \frac{f_3}{f_s} \cdot 2\pi = \frac{600}{1000} \cdot 2\pi = \frac{6\pi}{5} = 1.2\pi$$

2. (2 points) Explain whether any of the 3 sinusoids above are aliased. For each frequency that is aliased, assuming it was not stopped by a suitable antialias filter, calculate what frequency would be observed at the output of the system due to aliasing.

$|\omega_3| > \pi$ : it aliases  $+14k=1$

Method 1  $\rightarrow \frac{6\pi}{5} - k2\pi = \frac{-4\pi}{5} = \frac{f_{3a}}{f_s} \cdot 2\pi \rightarrow \frac{f_{3a}}{f_s} = \frac{-2}{5} \rightarrow f_{3a} = -400\text{Hz} \Rightarrow \boxed{400\text{Hz}}$

Method 2  $\rightarrow f_{3a} = f_3 - k \cdot f_s = 600 - 1000 = -400 \Rightarrow \boxed{400\text{Hz}}$   
( $+14k$ )

3. (4 points) Calculate the first 4 samples of the unit step response of  $y(n) - 0.3y(n-1) = x(n) + 5x(n-1) - 2x(n-2)$ . Recall that the step response is  $y(n)$  when  $x(n) = u(n)$ .

$y(n) = 0.3y(n-1) + x(n) + 5x(n-1) - 2x(n-2)$

n	x	y
0	1	$0.3 \times 0 + 1 + 5 \times 0 - 2 \times 0 = 1$
1	1	$0.3 \times 1 + 1 + 5 \times 1 - 2 \times 0 = 6.3$
2	1	$0.3 \times 6.3 + 1 + 5 \times 1 - 2 \times 1 = 5.89$
3	1	$0.3 \times 5.89 + 1 + 5 \times 1 - 2 \times 1 = 5.767$

Handwritten calculations for step response:

$$\begin{array}{r} 5.59 \\ \times 0.3 \\ \hline 1.677 \end{array}$$

$$\begin{array}{r} 6.3 \\ \times 0.3 \\ \hline 1.89 \end{array}$$

$$\begin{array}{r} 5.89 \\ \times 0.3 \\ \hline 1.767 \end{array}$$

4. (2 points) What is the vector of "a" or autoregressive or IIR (infinite impulse response) coefficients in the above equation? (Recall that the "b" or FIR coefficients correspond to a weighted sum of inputs.)

$$a = [1 \quad -0.3] = [a_0 \quad a_1]$$

$\uparrow$   
 $-\frac{1}{4}$