Name Answer

EE-3220-21 – Dr. Durant – Quiz 3
Winter 2013-'14, Week 3

1. (4 points) Calculate the first 4 samples of the unit ~~step~~ ~~impulse~~ step response of y(n) – 0.25 y(n-1) = x(n) – 2x(n-1) + 4x(n-2). Recall that the step response is y(n) when x(n) = u(n).

$y(n) = \frac{1}{4}y(n-1) + x(n) - 2x(n-1) + 4x(n-2)$

$y(0) = 0 + 1 - 0 + 0 = 1$ ✓

$y(1) = \frac{1}{4}\cdot 1 + 0 - 2\cdot 1 + 0 = \cancel{-1\frac{3}{4}} = \cancel{-1.75}$ $-\frac{3}{4} = -0.75$

$y(2) = -1\frac{3}{4}\cdot\frac{1}{4} + 0 - 0 + 4\cdot 1 = \cancel{3\frac{9}{16}} = \cancel{3.56..}$ $+2\frac{13}{16} = +2.81...$

$y(3) = \frac{1}{4}\cdot 3\frac{9}{16} + 0 - 0 + 0 = \cancel{\frac{57}{64}} = \cancel{0.89..}$ $+3\frac{45}{64} = +3.70...$

(-3/4) impulse, not step

2. (2 points) What are the autregressive or IIR (infinite impulse response) coefficients in the above equation? 1 & -0.25 (-0.25 alone is acceptable)

Specifically $a_0 = 1$ $a_1 = -0.25$

(-1/4) missing negative (-1/2) for FIR

3. (2 points) Calculate the discrete-time Fourier transform (DTFT) of the ~~impulse~~ ~~unit step~~ step response you calculated above. Recall that the DTFT is defined as $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n}$.

$X(e^{j\omega}) = 1 - \cancel{1\frac{3}{4}}e^{-j\omega} + \cancel{3\frac{9}{16}}e^{-j2\omega} + \cancel{\frac{57}{64}}e^{-j3\omega}$

$-\frac{3}{4}$ $+2\frac{13}{16}$ $+3\frac{45}{64}$

4. (2 points) How is the value of the DTFT at -ω related to its value at ω? Assume x is real. (You may just state the answer if you remember it from the book [we did not specifically discuss this], but you can also derive it from the DTFT definition similar to how we derived that $X(e^{j(\omega+2\pi)}) = X(e^{j\omega})$.)

$X(e^{-j\omega}) = \sum_n x(n)e^{+j\omega n} = \sum_n x(n)(e^{-j\omega n})^* = (\sum_n x(n)e^{-j\omega n})^* = X^*(e^{j\omega})$

evaluate @ $-\omega$

conjugate

since x(n) multiply doesn't change phase & sum of conjugates is conjugate of sum

Answer: $\boxed{\text{It is the conjugate.}}$

(-3/4) use principle

(-1/2) equal mag (wrong/unspec. ϕ)

Name *Answers*

EE-3220-41 – Dr. Durant – Quiz 3
Winter 2013-'14, Week 3

1. (4 points) Calculate the first 4 samples of the unit step response of y(n) – 0.5 y(n-1) = 2x(n) – x(n-2). Recall that the step response is y(n) when x(n) = u(n).

$y(n) = \frac{1}{2} y(n-1) + 2u(n) - u(n-2)$ (Solve for $y(n)$, set $x(n)=u(n)$)

0-state response

$y(0) = \frac{1}{2}\cdot 0 + 2u(0) - u(-2) = 0+2-0=2$

$y(1) = \frac{1}{2}\cdot 2 + 2u(1) - u(-1) = 1+2-0=3$

$y(2) = \frac{1}{2}\cdot 3 + 2u(2) - u(0) = 1\frac{1}{2}+2-1=2\frac{1}{2}$

$y(3) = \frac{1}{2}\cdot 2\frac{1}{2} + 2u(3) - u(1) = 1\frac{1}{4}+2-1=2\frac{1}{4}$

2. (2 points) What are the moving average or FIR (finite impulse response) coefficients in the above equation?

coefficients of x : 2 & -1
specifically, $b = [2 \;\; 0 \;\; -1]$

3. (2 points) Calculate the discrete-time Fourier transform (DTFT) of the ~~impulse~~ unit step response you calculated above. Recall that the DTFT is defined as $X(e^{j\omega}) \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n}$

$$Y(e^{j\omega}) = 2 + 3e^{-j\omega} + 2\tfrac{1}{2} e^{-j2\omega} + 2\tfrac{1}{4} e^{-j3\omega}$$

4. (2 points) How is the value of the DTFT at ω+2π related to its value at ω? (You may just state the answer if you remember it, but you can also derive it from the DTFT definition.)

They are equal.

Because:

$$X(e^{j(\omega+2\pi)}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j(\omega+2\pi)n} = \sum_n x(n) e^{-j\omega n} \underbrace{e^{-j2\pi n}}_{=1 \text{ since } n\in\mathbb{Z} \text{ (n is an integer)}} = \sum_n x(n) e^{-j\omega n} = X(e^{j\omega})$$