

EE-3220-11 - Dr. Durant - Quiz 3  
Spring 2015, Week 3

1. (2 points) Let  $f_s = 2000$  Hz,  $f_1 = 0$  Hz,  $f_2 = 500$  Hz, and  $f_3 = 1500$  Hz. **Calculate** the digital frequencies,  $\omega_n$ , for each frequency,  $f_n$ , for  $f_1$  through  $f_3$ . Recall that the digital frequency is how many radians a sinusoid moves through between samples. For example, if a signal is sampled 10 times per period, its digital frequency is  $2\pi/10$ . Do **not** make any adjustments for aliasing.

$$\begin{aligned} \omega_1 &= \frac{f_1}{f_s} \cdot 2\pi = \frac{0}{2000} \cdot 2\pi = 0 \text{ radians/sample} \\ \omega_2 &= \frac{500}{2000} \cdot 2\pi = \frac{\pi}{2} \\ \omega_3 &= \frac{1500}{2000} \cdot 2\pi = \frac{3\pi}{2} \end{aligned}$$

2. (2 points) **Explain** whether any of the 3 sinusoids above are aliased. For each frequency that is aliased, assuming it was not stopped by a suitable antialias filter, **calculate** what frequency would be observed at the output of the system due to aliasing.

$|\omega_3| > \pi \therefore$  it is aliased

Digital signals are periodic in frequency, so  $\omega_3^* = \omega_3 + 2\pi k$  gives some samples for all integers  $k$  ( $k \in \mathbb{Z}$ ). Choose  $k = -1$ .  
 $\omega_3^* = \frac{3\pi}{2} - 2\pi = -\frac{\pi}{2}$ . Now,  $|\omega_3^*| \leq \pi$  so it is the reconstructed frequency. (Since signal is real, the negative freq. component also exists, so  $+\pi/2$  is also right.)

3. (4 points) **Calculate** the first 3 samples of the unit step response of  $y(n) - 0.5y(n-1) = x(n) - 3x(n-1) + 4x(n-2)$ . Recall that the step response is  $y(n)$  when  $x(n) = u(n)$ .

$$\begin{aligned} y(n) &= \frac{1}{2}y(n-1) + x(n) - 3x(n-1) + 4x(n-2) \\ y(0) &= \frac{1}{2} \cdot 0 + 1 - 0 + 0 = 1 \\ y(1) &= \frac{1}{2} \cdot 1 + 1 - 3 \cdot 1 + 0 = -1.5 \\ y(2) &= \frac{1}{2} \cdot (-1.5) + 1 - 3 \cdot 1 + 4 \cdot 1 = 1.25 \end{aligned}$$

4. (2 points) What is the vector of "a" or autoregressive or IIR (infinite impulse response) coefficients in the above equation? (Recall that the "b" or FIR coefficients correspond to a weighted sum of inputs.)

$$a = [1 \quad -0.5] = [a_0 \quad a_1]$$

↑  
negative, keep all a's in standard form/left hand side