

EE-3221 - Dr. Durant - Quiz 2
Winter 2020-'21, Week 2

This is an open-book quiz. You should find Table 5-6 and 5-7 especially helpful.

1. (3 points) Let $s(t) = \delta(t) + \delta(t-1) + \delta(t-2)$. Find $S(\omega)$, the Fourier Transform of $s(t)$, using tables.
2. (3 points) Let $x(t) = \cos(\pi/2 \times t)$. Find $X(\omega)$, the Fourier Transform of $x(t)$, using tables.
3. (4 points) Using tables and **without** evaluating the convolution integral, calculate the convolution of the 2 FTs you found above, $S(\omega) * X(\omega)$. Hint: Convolution in frequency property, Table 5-7.12.

① S-6.1: $\delta(t) \Leftrightarrow 1$
 S-7.4: $x(t-t_0) \Leftrightarrow e^{-j\omega t_0} X(\omega)$
 $\therefore S(\omega) = 1 + e^{-j\omega} + e^{-2j\omega}$

② S-6.8: $\cos(\omega_0 t) \Leftrightarrow \pi [\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$
 $\therefore X(\omega) = \pi [\delta(\omega - \frac{\pi}{2}) + \delta(\omega + \frac{\pi}{2})]$

③ S-7.12: $x_1(t) x_2(t) \Leftrightarrow \frac{1}{2\pi} X_1(\omega) * X_2(\omega)$
 $\therefore 2\pi s(t) x(t) \Leftrightarrow S(\omega) * X(\omega)$

\therefore Find $2\pi s(t) x(t) = 2\pi (\delta(t) + \delta(t-1) + \delta(t-2)) \cos(\frac{\pi}{2} t)$
 scale impulse area per \uparrow
 $= 2\pi (\underbrace{\cos(0)}_1 \delta(t) + \underbrace{\cos(\frac{\pi}{2})}_0 \delta(t-1) + \underbrace{\cos(\frac{\pi}{2} \cdot 2)}_{-1} \delta(t-2))$
 $= 2\pi (\delta(t) - \delta(t-2))$

So, take the FT, which is a variation on #1

$2\pi (\delta(t) - \delta(t-2)) \Leftrightarrow 2\pi (1 - e^{-2j\omega}) = S(\omega) * X(\omega)$