Name Answer

EE-3221-11 – Dr. Durant – Quiz 2
Winter 2017-‘18, Week 2

1. (2 points) Consider a 3 Hz cosine wave, $x(t) = \cos(2\pi \times 3 \times t)$. Plot the magnitude of $X(\Omega)$ for this signal.

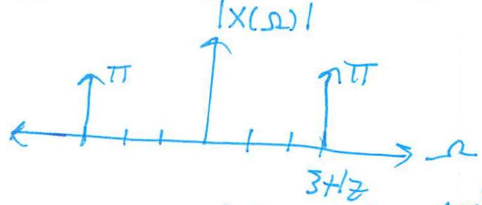
2. (2 points) Plot the magnitude of the sampled spectrum of the above signal, $X_s(\Omega)$ for fs = 5 Hz. Note that the Nyquist criterion is not satisfied, so aliasing occurs.

3. (2 points) Based on your previous plot, what aliased frequency appears?

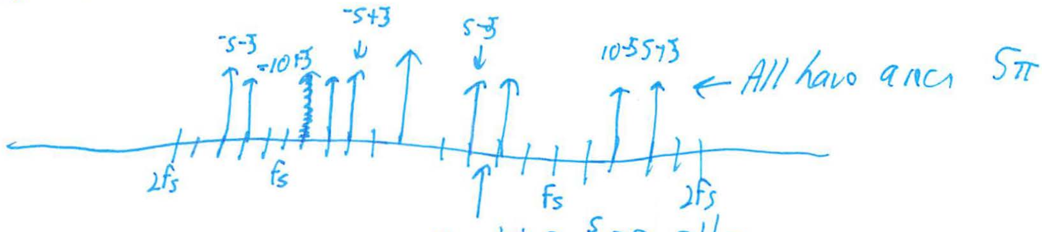
4. (2 points) Derive an expression for the sampled signal x(n) using the given fs; this will not have impulses, just finite values.

5. (2 points) Explain why it is impossible to exactly represent a square wave in a DSP system. (Hint: Table 5.2, Line 12)

① $X(\Omega) = \pi[\delta(\Omega - 3\,Hz) + \delta(-\Omega + 3\,Hz)]$



② Scale by $\frac{1}{T_s} = 5$ (not required for full credit).
Images @ k·5Hz ∴ 5k±3



③ 2Hz (∓5±3)

④ $t = nT_s = n/5$ $\quad x(n) = \boxed{\cos\left(2\pi \times 3 \times \frac{n}{5}\right) = \cos\left(\frac{6\pi n}{5}\right)}$ ← expected answer

$= \cos\left(\frac{6\pi}{5} \times n\right) = \cos\left(\left(\frac{6\pi}{5} - 2\pi\right)n\right) = \cos\left(-\frac{4\pi n}{5}\right) = \cos\left(\frac{4}{\ldots}\right.$ ← 4Hz ali[as]

⑤ A sharp edge/instantaneous output change requires infinitely high frequencies, violating Nyquist.

Name *Answers*

EE-3221-41 – Dr. Durant – Quiz 2
Winter 2017-'18, Week 2

1. (2 points) Consider an 8 Hz sine wave, $x(t) = \sin(2\pi \times 8 \times t)$. Plot the magnitude of $X(\Omega)$ for this signal.

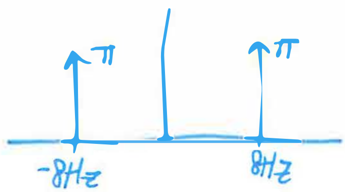
2. (2 points) Plot the magnitude of the sampled spectrum of the above signal, $X_s(\Omega)$ for fs = 10 Hz. Note that the Nyquist criterion is not satisfied, so aliasing occurs.

3. (2 points) Based on your previous plot, what aliased frequency appears?

4. (2 points) Derive an expression for the sampled signal x(n) using the given fs; this will not have impulses, just finite values.

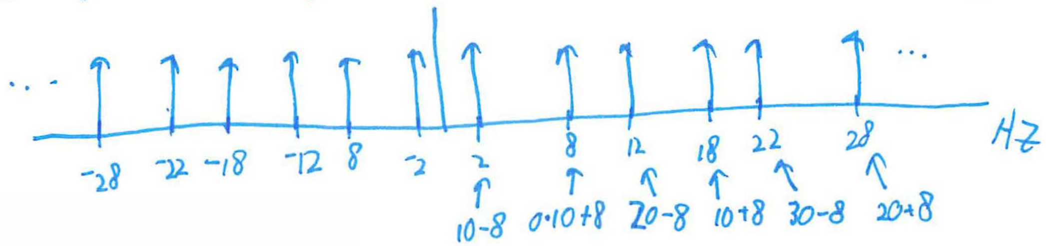
5. (2 points) Explain why it is impossible to exactly represent a one-sided exponential decay in a DSP system. (Hint: Table 5.2, Line 7)

(1) $X(\Omega) = -j\pi[\delta(\Omega - 8Hz) - \delta(\Omega + 8Hz)]$



(2) Copy spectrum every kfs. ∴ $k \cdot 10 \pm 8$

All have area $\pi \cdot \frac{1}{T_s} = 10\pi$
(not req'd. for full credit)



(3) 2 Hz (Nyquist is $10/2 = 5$ Hz)

(4) $t = nT_s = n/f_s = n/10$ $x(n) = \sin(2\pi \cdot 8 \cdot \frac{n}{10}) = \sin(1.6\pi n)$ ← expected answer

$= \sin((1.6-2)\pi n) = \sin(-0.4\pi n) = -\sin(\frac{4\pi}{10} n) = -\sin(\frac{2 \cdot 2 \dots}{10 \dots}$

aliased f=2, fs

(5) $X(\Omega) = \frac{A}{j\Omega + a}$ ∴ $|X(\Omega)| > 0$ for large Ω, violates Nyquist

Table 5.1

Basic Properties of Fourier Transform

	Time Domain	Frequency Domain
Signals and constants	$x(t), y(t), z(t), \alpha, \beta$	$X(\Omega), Y(\Omega), Z(\Omega)$
Linearity	$\alpha x(t) + \beta y(t)$	$\alpha X(\Omega) + \beta Y(\Omega)$
Expansion/contraction in time	$x(\alpha t), \alpha \neq 0$	$\frac{1}{\|\alpha\|} X\left(\frac{\Omega}{\alpha}\right)$
Reflection	$x(-t)$	$X(-\Omega)$
Parseval's energy relation	$E_x = \int_{-\infty}^{\infty} \|x(t)\|^2 dt$	$E_x = \frac{1}{2\pi} \int_{-\infty}^{\infty} \|X(\Omega)\|^2 d\Omega$
Duality	$X(t)$	$2\pi x(-\Omega)$
Time differentiation	$\frac{d^n x(t)}{dt^n}, n \geq 1$, integer	$(j\Omega)^n X(\Omega)$
Frequency differentiation	$-jtx(t)$	$\frac{dX(\Omega)}{d\Omega}$
Integration	$\int_{-\infty}^{t} x(t')dt'$	$\frac{X(\Omega)}{j\Omega} + \pi X(0)\delta(\Omega)$
Time shifting	$x(t - \alpha)$	$e^{-j\alpha\Omega} X(\Omega)$
Frequency shifting	$e^{j\Omega_0 t} x(t)$	$X(\Omega - \Omega_0)$
Modulation	$x(t) \cos(\Omega_c t)$	$0.5[X(\Omega - \Omega_c) + X(\Omega + \Omega_c)]$
Periodic signals	$x(t) = \sum_k X_k e^{jk\Omega_0 t}$	$X(\Omega) = \sum_k 2\pi X_k \delta(\Omega - k\Omega_0)$
Symmetry	$x(t)$ real	$\|X(\Omega)\| = \|X(-\Omega)\|$
		$\angle X(\Omega) = -\angle X(-\Omega)$
Convolution in time	$z(t) = [x * y](t)$	$Z(\Omega) = X(\Omega)Y(\Omega)$
Windowing/Multiplication	$x(t)y(t)$	$\frac{1}{2\pi}[X * Y](\Omega)$
Cosine transform	$x(t)$ even	$X(\Omega) = \int_{-\infty}^{\infty} x(t)\cos(\Omega t)dt$, real
Sine transform	$x(t)$ odd	$X(\Omega) = -j\int_{-\infty}^{\infty} x(t)\sin(\Omega t)dt$, imaginary

Table 5.2

Fourier Transform Pairs

	Function of Time	Function of Ω
(1)	$\delta(t)$	1
(2)	$\delta(t - \tau)$	$e^{-j\Omega\tau}$
(3)	$u(t)$	$\frac{1}{j\Omega} + \pi\delta(\Omega)$
(4)	$u(-t)$	$\frac{-1}{j\Omega} + \pi\delta(\Omega)$
(5)	$\text{sign}(t) = 2[u(t) - 0.5]$	$\frac{2}{j\Omega}$
(6)	$A, -\infty < t < \infty$	$2\pi A\delta(\Omega)$
(7)	$Ae^{-at}u(t), a > 0$	$\frac{A}{j\Omega + a}$
(8)	$Ate^{-at}u(t), a > 0$	$\frac{A}{(j\Omega + a)^2}$
(9)	$e^{-a\|t\|}, a > 0$	$\frac{2a}{a^2 + \Omega^2}$
(10)	$\cos(\Omega_0 t), -\infty < t < \infty$	$\pi[\delta(\Omega - \Omega_0) + \delta(\Omega + \Omega_0)]$
(11)	$\sin(\Omega_0 t), -\infty < t < \infty$	$-j\pi[\delta(\Omega - \Omega_0) - \delta(\Omega + \Omega_0)]$
(12)	$p(t) = A[u(t + \tau) - u(t - \tau)], \tau > 0$	$2A\tau \frac{\sin(\Omega\tau)}{\Omega\tau}$
(13)	$\frac{\sin(\Omega_0 t)}{\pi t}$	$P(\Omega) = u(\Omega + \Omega_0) - u(\Omega - \Omega_0)$
(14)	$x(t)\cos(\Omega_0 t)$	$0.5[X(\Omega - \Omega_0) + X(\Omega + \Omega_0)]$