Name *Answers*

EE-3221-11 – Dr. Durant – Quiz 1
Winter 2017-'18, Week 1

1. (2 points) A signal is sampled at 20 Hz. What is the maximum frequency that the system can process without aliasing?

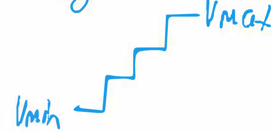
2. (2 points) A signal is quantized to levels ranging from a minimum of -2 V to a maximum of +2 V. The step between the levels is ¼ V. How many levels are there in this quantization scheme?

3. (3 points) Draw the basic DSP system block diagram showing the 5 components in series.

① Nyquist freq. $= \frac{F_s}{2} = \frac{20\,Hz}{2} = \boxed{10\ Hz}$

② Steps $= \frac{V_{max} - V_{min}}{V_{step}} = \frac{2 - -2}{1/4} = 16$

E.g. 4 levels, 3 steps



Levels = steps + 1 = 17

③ $x(t) \rightarrow$ Anti-alias filter $\rightarrow$ A/D $\rightarrow$ DSP $\rightarrow$ D/A $\rightarrow$ Reconstruction Filter $\rightarrow y(t)$

Table 5.1

Basic Properties of Fourier Transform

	Time Domain	Frequency Domain
Signals and constants	$x(t), y(t), z(t), \alpha, \beta$	$X(\Omega), Y(\Omega), Z(\Omega)$
Linearity	$\alpha x(t) + \beta y(t)$	$\alpha X(\Omega) + \beta Y(\Omega)$
Expansion/contraction in time	$x(\alpha t), \alpha \neq 0$	$\frac{1}{\lvert\alpha\rvert} X\left(\frac{\Omega}{\alpha}\right)$
Reflection	$x(-t)$	$X(-\Omega)$
Parseval's energy relation	$E_x = \int_{-\infty}^{\infty} \lvert x(t)\rvert^2 dt$	$E_x = \frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert X(\Omega)\rvert^2 d\Omega$
Duality	$X(t)$	$2\pi x(-\Omega)$
Time differentiation	$\frac{d^n x(t)}{dt^n}, n \geq 1$, integer	$(j\Omega)^n X(\Omega)$
Frequency differentiation	$-jtx(t)$	$\frac{dX(\Omega)}{d\Omega}$
Integration	$\int_{-\infty}^{t} x(t')dt'$	$\frac{X(\Omega)}{j\Omega} + \pi X(0)\delta(\Omega)$
Time shifting	$x(t-\alpha)$	$e^{-j\alpha\Omega}X(\Omega)$
Frequency shifting	$e^{j\Omega_0 t}x(t)$	$X(\Omega - \Omega_0)$
Modulation	$x(t)\cos(\Omega_c t)$	$0.5[X(\Omega - \Omega_c) + X(\Omega + \Omega_c)]$
Periodic signals	$x(t) = \sum_k X_k e^{jk\Omega_0 t}$	$X(\Omega) = \sum_k 2\pi X_k \delta(\Omega - k\Omega_0)$
Symmetry	$x(t)$ real	$\lvert X(\Omega)\rvert = \lvert X(-\Omega)\rvert$
		$\angle X(\Omega) = -\angle X(-\Omega)$
Convolution in time	$z(t) = [x * y](t)$	$Z(\Omega) = X(\Omega)Y(\Omega)$
Windowing/Multiplication	$x(t)y(t)$	$\frac{1}{2\pi}[X * Y](\Omega)$
Cosine transform	$x(t)$ even	$X(\Omega) = \int_{-\infty}^{\infty} x(t)\cos(\Omega t)dt$, real
Sine transform	$x(t)$ odd	$X(\Omega) = -j\int_{-\infty}^{\infty} x(t)\sin(\Omega t)dt$, imaginary

Table 5.2

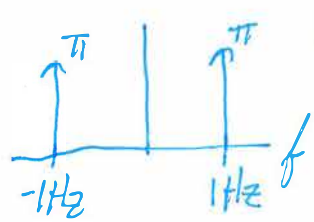
Fourier Transform Pairs

	Function of Time	Function of Ω
(1)	$\delta(t)$	1
(2)	$\delta(t-\tau)$	$e^{-j\Omega\tau}$
(3)	$u(t)$	$\frac{1}{j\Omega} + \pi\delta(\Omega)$
(4)	$u(-t)$	$\frac{-1}{j\Omega} + \pi\delta(\Omega)$
(5)	$\text{sign}(t) = 2[u(t) - 0.5]$	$\frac{2}{j\Omega}$
(6)	$A, -\infty < t < \infty$	$2\pi A\delta(\Omega)$
(7)	$Ae^{-at}u(t), a > 0$	$\frac{A}{j\Omega + a}$
(8)	$Ate^{-at}u(t), a > 0$	$\frac{A}{(j\Omega + a)^2}$
(9)	$e^{-a\lvert t\rvert}, a > 0$	$\frac{2a}{a^2 + \Omega^2}$
(10)	$\cos(\Omega_0 t), -\infty < t < \infty$	$\pi[\delta(\Omega - \Omega_0) + \delta(\Omega + \Omega_0)]$
(11)	$\sin(\Omega_0 t), -\infty < t < \infty$	$-j\pi[\delta(\Omega - \Omega_0) - \delta(\Omega + \Omega_0)]$
(12)	$p(t) = A[u(t+\tau) - u(t-\tau)], \tau > 0$	$2A\tau\frac{\sin(\Omega\tau)}{\Omega\tau}$
(13)	$\frac{\sin(\Omega_0 t)}{\pi t}$	$P(\Omega) = u(\Omega + \Omega_0) - u(\Omega - \Omega_0)$
(14)	$x(t)\cos(\Omega_0 t)$	$0.5[X(\Omega - \Omega_0) + X(\Omega + \Omega_0)]$

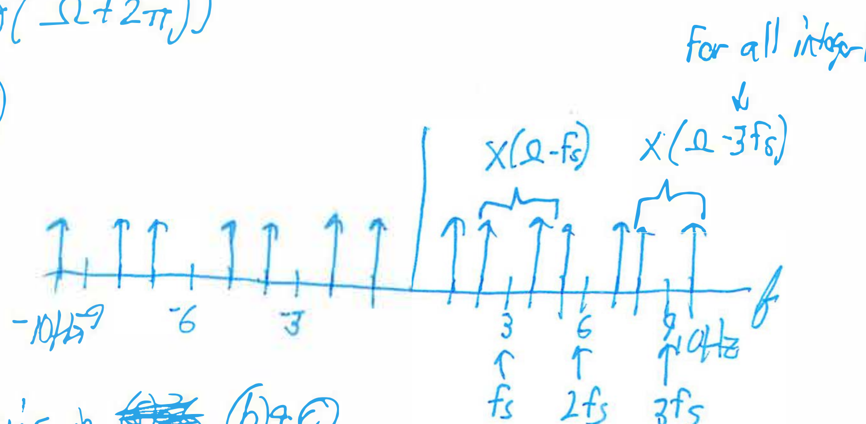
4. (3 points) Fourier transforms
   a. Find the Fourier transform, X(Ω), of x(t) = cos(2πt). (See Table 5.2.)
   b. Plot the magnitude of this Fourier transform. Label the frequency axis in hertz. Indicate the area inside any impulses in the graph
   c. Recall that ideal sampling involves sampling a signal by multiplying it by an impulse train (series of impulses separated by Ts, the sampling time). This results in a signal that is 0 almost everywhere but has impulses having area proportional to the input voltage every Ts seconds. This sampling causes the Fourier transform of x(t) to be replicated every fs. With this in mind, sketch the magnitude of the Fourier transform of x(t) when it is sampled at a frequency of 3 Hz.

(a) $X(\Omega) = \pi\left(\delta(\Omega - 2\pi) + \delta(\Omega + 2\pi)\right)$

(b)



(c)



Note: Ok to not label y-axis in (b)+(c).
Technically it is V/Hz = V·s, if x(t) is in volts.

Name Answer

EE-3221-41 – Dr. Durant – Quiz 1
Winter 2017-'18, Week 1

1. (2 points) A signal contains energy at various frequencies up to 30 Hz. What is the minimum sampling frequency that will not cause aliasing?

2. (2 points) A signal is quantized to 4 (very few!) levels. The minimum level is -1 V and the maximum level is 1 V. What is the step between the levels? (Hint: be careful not to confuse the number of levels with the number of steps; it may help to make a sketch.)

3. (3 points) Draw the basic DSP system block diagram showing the 5 components in series.

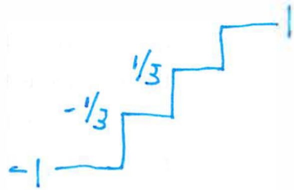
① $f_{Nyquist} = \frac{F_s}{2}$

$30\,Hz = \frac{F_s}{2}$

$F_s = 60\,Hz$

② $Steps = levels - 1 = 4 - 1 = 3$

$\Delta V = \frac{V_{max} - V_{min}}{Steps} = \frac{(1 - -1)V}{3} = \boxed{2/3\ V}$



③

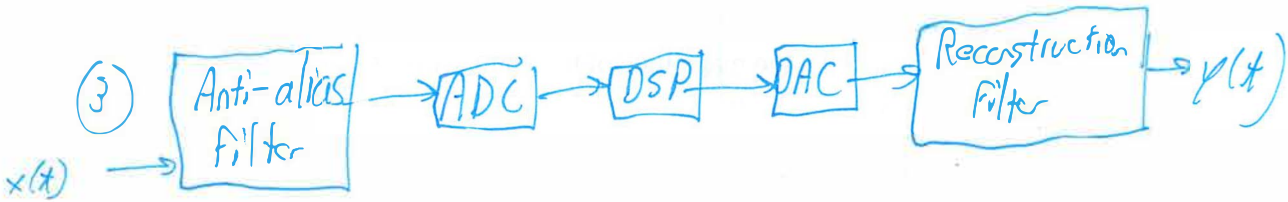


Table 5.1

Basic Properties of Fourier Transform

	Time Domain	Frequency Domain
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Parseval's energy relation	$E_x = \int_{-\infty}^{\infty} \lvert x(t)\rvert^2 dt$	$E_x = \frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert X(\Omega)\rvert^2 d\Omega$
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Time differentiation	$\frac{d^n x(t)}{dt^n}, n \geq 1$, integer	$(j\Omega)^n X(\Omega)$
Frequency differentiation	$-jtx(t)$	$\frac{dX(\Omega)}{d\Omega}$
Integration	$\int_{-\infty}^{t} x(t')dt'$	$\frac{X(\Omega)}{j\Omega} + \pi X(0)\delta(\Omega)$
Time shifting	$x(t - \alpha)$	$e^{-j\alpha\Omega} X(\Omega)$
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Periodic signals	$x(t) = \sum_k X_k e^{jk\Omega_0 t}$	$X(\Omega) = \sum_k 2\pi X_k \delta(\Omega - k\Omega_0)$
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Table 5.2

Fourier Transform Pairs

	Function of Time	Function of Ω
(1)	$\delta(t)$	1
(2)	$\delta(t - \tau)$	$e^{-j\Omega\tau}$
(3)	$u(t)$	$\frac{1}{j\Omega} + \pi\delta(\Omega)$
(4)	$u(-t)$	$\frac{-1}{j\Omega} + \pi\delta(\Omega)$
(5)	$\text{sign}(t) = 2[u(t) - 0.5]$	$\frac{2}{j\Omega}$
(6)	$A, -\infty < t < \infty$	$2\pi A\delta(\Omega)$
(7)	$Ae^{-at}u(t), a > 0$	$\frac{A}{j\Omega + a}$
(8)	$Ate^{-at}u(t), a > 0$	$\frac{A}{(j\Omega + a)^2}$
(9)	$e^{-a\lvert t\rvert}, a > 0$	$\frac{2a}{a^2 + \Omega^2}$
(10)	$\cos(\Omega_0 t), -\infty < t < \infty$	$\pi[\delta(\Omega - \Omega_0) + \delta(\Omega + \Omega_0)]$
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(12)	$p(t) = A[u(t + \tau) - u(t - \tau)], \tau > 0$	$2A\tau\frac{\sin(\Omega\tau)}{\Omega\tau}$
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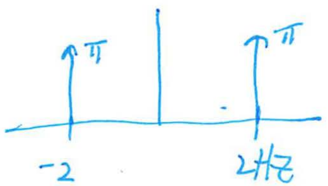
4. (3 points) Fourier transforms
   a. Find the Fourier transform, $X(\Omega)$, of $x(t) = \sin(4\pi t)$. (See Table 5.2.)
   b. Plot the magnitude of this Fourier transform. Label the frequency axis in hertz. Indicate the area inside any impulses in the graph
   c. Recall that ideal sampling involves sampling a signal by multiplying it by an impulse train (series of impulses separated by Ts, the sampling time). This results in a signal that is 0 almost everywhere but has impulses having area proportional to the input voltage every Ts seconds. This sampling causes the Fourier transform of x(t) to be replicated every fs. With this in mind, sketch the magnitude of the Fourier transform of x(t) when it is sampled at a frequency of 5 Hz.

(4) (a) $X(\Omega) = -j\pi[\delta(\Omega - 4\pi) - \delta(\Omega + 4\pi)]$

(b)



(c)