9.2 (a) $x(n) = \cos(0.7\pi n)$

$\omega = 0.7\pi = 2\pi \frac{k}{N}$

$\frac{k}{N} = \frac{7}{20}$ $\therefore N = $ multiple of 20

$N_0 = $ minimum $N = \boxed{20}$

(b) ~~Regardless of T_s, Nyquist is satisfied since $\omega < \pi$~~

~~But, in seconds,~~ $\omega = 2\pi \frac{f}{f_s} = 2\pi f T_s$

$0.7\pi = 2\pi f T_s$

$0.35 = f T_s$

~~$T_s = 0.7s$~~ $f = 0.35/T_s = 0.35/0.7 = 1/2 = 0.5\,Hz$

Wheeps.

(b) redefines x...

$x(t) = \cos(\pi t)$ $t = nT_s = 0.7n$ $\therefore$ $x(n) = \cos(0.7\pi t) \rightarrow$ $\boxed{\text{Samples same as a}}$

$\omega = 0.7\pi < \pi \therefore$ $\boxed{\text{Nyquist satisfied,}}$ no aliasing

Or, $f_s = \frac{1}{T_s} = \frac{10}{7} = 1\frac{3}{7}\,Hz$ $\Omega = \pi$ $f = \frac{\Omega}{2\pi} = \frac{1}{2} = 0.5\,Hz$

$f < f_s/2 \therefore$ no aliasing

(c) Begin by satisfying Nyquist

$\Omega > \frac{\Omega_s}{2}$ OR $f < \frac{f_s}{2}$

$0.5 < f_s/2$

$f_s > 1\,Hz$ $\rightarrow$ $T_s = \frac{1}{f_s}$

$T_s < 1\,s$

To closely resemble, we want

$\boxed{T_s \ll 1\,s}$

Note: Book's answer also assumes the resulting sampled signal must be periodic, but problem didn't state this.

Problem 9.3

(a) (i) $x(n) = 2\cos(\pi n - \frac{\pi}{2})$ $\quad \omega = \pi = 2\pi \frac{k}{N}$ $\quad \frac{k}{N} = \frac{1}{2}$ $\therefore \boxed{N_0 = 2}$ (periodic)

(ii) $y(n) = \sin(n - \pi/2)$ $\quad \omega = 1 = 2\pi \frac{k}{N}$ $\quad \frac{k}{N} = \frac{1}{2\pi}$, No integer solution, $\boxed{\text{not periodic}}$

(iii) $z(n) = x(n) + y(n)$ $\quad y$ is not periodic + is not cancelled $\therefore$ $\boxed{\text{not periodic}}$

(iv) $v(n) = \sin(\frac{3\pi}{2} n)$ $\quad \omega = \frac{3\pi}{2} = 2\pi \frac{k}{N}$, $\frac{k}{N} = \frac{3}{4}$, $\boxed{N_0 = 4}$

(b) $x_1(n)$ has fund. period $N_1 = 4$
$y_1(n)$ " " " $N_2 = 6$

(i) $z_1(n) = x_1(n) + y_1(n)$ $\quad N_0 = \text{lcm}(N_1, N_2) = \boxed{12}$
(12 is the smallest # that is a period of all components, setting them simultaneously back to 0-phase / initial phase)

(ii) $v_1(n) = x_1(n) y_1(n)$ $\quad$ Again, $N_0 = \text{lcm}(N_1, N_2) = 12$

Check that N_0 is a period:

$v_1(n+12) = x_1(n+12)\, y_1(n+12)$
$= x_1(n + 3N_1)\, y_1(n + 2N_2)$
$= x_1(n)\, y_1(n) = v_1(n)$

(iii) $w_1(n) = x_1(2n)$ $\quad$ (downsample, discard odd half of samples)

Period is cut in half, $N_0 = N_1/2 = \boxed{2}$

Check: $w_1(n+2) = x_1(2(n+2)) = x_1(\underbrace{2n+4}_{\text{period } 4}) = x_1(2n)$ ✓

Challenge: - Show this only works if N_1 is even.
- Show if N_1 is odd, then $N_0 = N_1$.

Interesting result for sampled periodic signals.