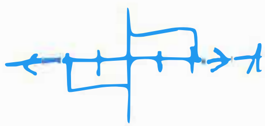
5.5 Using $x(t) = u(t+1) - u(t-1) \leftrightarrow X(\Omega) = \frac{2\sin(\Omega)}{\Omega} = 2\,\mathrm{sinc}(\Omega)$

indicated properties needed to take F.T. of...

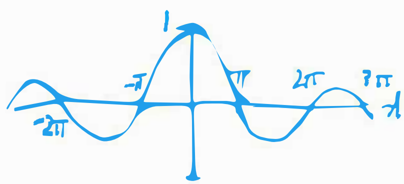
(a) $x_1(t) = -u(t+2) + 2u(t) - u(t-2)$

use time shift property (twice)
(& linearity to multiply -1)
$x(t-\alpha) \leftrightarrow e^{-j\alpha\Omega} X(\Omega)$



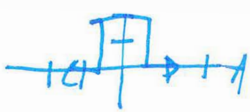
(b) $x_2(t) = 2\frac{\sin(t)}{t} = 2\,\mathrm{sinc}(t)$

use duality of the Fourier transform
$X(t) \leftrightarrow 2\pi x(-\Omega)$



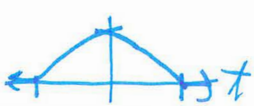

(c) $x_3(t) = 2[u(t+\frac{1}{2}) - u(t-\frac{1}{2})]$

use scaling property
(time-scaling)
(expansion/contraction in time)
$x(\alpha t) \leftrightarrow \frac{1}{|\alpha|} X(\frac{\Omega}{\alpha})$
$\alpha \neq 0$



(d) $x_4(t) = \cos(0.5\pi t)[u(t+1) - u(t-1)]$

use modulation property
$x(t)\cos(\Omega_c t) \leftrightarrow 0.5[X(\Omega - \Omega_c) + X(\Omega + \Omega_c)]$



(e) $x_5(t) = X(t) = 2\,\mathrm{sinc}(t)$

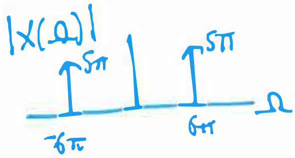
same sketch & property as (b)

$f$ in hertz

$$x(t) = 5\cos(6\pi t) = 5\cos(2\pi \cdot \underset{\uparrow}{3} \cdot t)$$

(a) $X(\Omega) = 5\pi\left(\delta(\Omega - 6\pi) + \delta(\Omega + 6\pi)\right)$

(b)

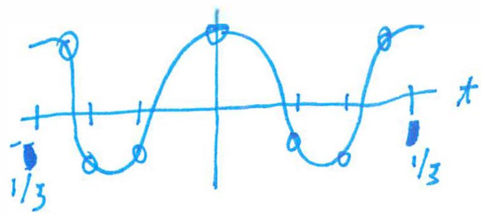



(c) $f_s = 10\text{ Hz}$ $\quad t = n \cdot T_s = n \cdot \frac{1}{10\text{ Hz}} = n \cdot \frac{1}{10}\text{ s}$

$$x(n) = 5\cos\left(6\pi \cdot \frac{n}{10}\right) = 5\cos\left(\frac{3\pi}{5} n\right)$$

Graph <u>not</u> required, but...

$\frac{3\pi}{5} \frac{\text{radians}}{\text{sample}} = 108°/\text{sample}$




(d) $f = \frac{\Omega}{2\pi} = \frac{6\pi}{2\pi} = 3\text{ Hz}$

$f \overset{?}{\leq} \frac{f_s}{2}$

$3 \leq \frac{10}{2} = 5$, yes ∴ Nyquist satisfied, no aliasing

(e) $X_s(\Omega) = 5\pi \sum_n \left(\delta(\Omega - n\Omega_s) + \delta(\Omega + n\Omega_s)\right)$

$X_s(f) = 5\pi \sum_n \left(\delta(f - nf_s) + \delta(f + nf_s)\right)$