Milwaukee School of Engineering

Electrical Engineering and Computer Science Department

EE-3221 – Final Exam – Dr. Durant

Monday 21 February 2022

May use textbook (PDF or printed), 8½" × 11" note sheet, and calculator.

Good luck!

Name: Answers

Page 2:	(23 points)	______
Page 3:	(25 points)	______
Page 4:	(16 points)	______
Page 5:	(21 points)	______
Page 6:	(15 points)	______
Total:	(100 points)	______

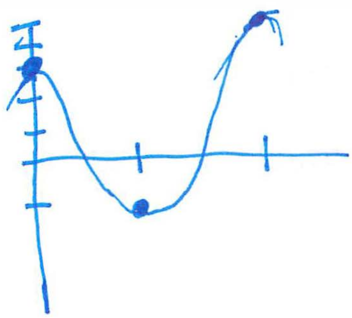
23

1. (10 points) Illustrate the purpose of the reconstruction filter in a DSP system by sketching its input and output for the samples y[n] = [3 -1 5].

2. (8 points) An analog signal with energy in the frequency range 0-20 kHz undergoes the following operations in order: 1) lowpass filtering at 10 kHz, 2) sampling at 16 kHz. Aliasing occurs at the higher frequencies. What ***range of frequencies*** are ***not aliased*** and thus could be recovered in a lowpass version of the signal with an even lower cutoff frequency than the original 10 kHz?

3. (5 points) What is the period of $x(n) = \cos(9\pi n/25)$? Show your work.

(1)



(2) Alias down to $16-10 = 6\,kHz \rightarrow 0 \rightarrow 6\,kHz$ not aliased

Equivalently: $Ny = \frac{16}{2} = 8\,kHz$, Alias $8 \rightarrow 10 \Rightarrow 8 \rightarrow 6$

(−2) 0−8 w figure/logic, but some error

(3) $\Omega = \frac{9\pi}{25}$

$N_0 \Omega = 2\pi k$

$\frac{9\pi}{25} N_0 = 2\pi k$

$9 N_0 = 50k$, irreducible

$\therefore \boxed{N_0 = 50}$

25

4. (10 points) Calculate the convolution [3 -6 4] * [7 2] directly using the convolution formula. Show your work.

5. (10 points) Recalculate the convolution using the DFT/IDFT method. It is suggested that you use the FFT that we learned in class (radix-2 decimation-in-time FFT).

6. (5 points) Discuss whether your convolution results are consistent with the ***area property***.

(4)

$$\begin{array}{cccc} 21 & -42 & 28 & \\ & 6 & -12 & 8 \\ \hline [21 & -36 & 16 & 8] \end{array}$$

(5) x

n	x				h		
0	3	7	1	7	7	9	
2	4	-1	$-1+6j$		7	$7-2j$	
1	-6	-6	13	2	2	5	
3		-6	$-1-6j$		2	$7+2j$	

(-7) $N_0 = \{2, 3\}$

$Y[k] = X[k] \cdot H[k] = [9 \;\; 5+44j \;\; 65 \;\; 5-44j]$ (-3) stop here, (-1) claim this is $y[n]$

k				/N	
0	9	74	84	→	21
2	65	-56	-144	→	-36
1	$5+44j$	10	64	→	16
3	$5-44j$	$88j$	32	→	8

matches #4 ✓

(6) $A_x = \Sigma x = 3-6+4 = 1$

$A_h = \Sigma h = 7+2 = 9$

$A_y = \Sigma y = 21-36+16+8 = 9$

$A_y = A_x \cdot A_h = 1 \cdot 9 = 9$ ✓

Yes, A_y by both methods agrees.

16

7. (6 points) For N = 2^{11} = 2048, calculate the number of multiplies needed for (a) directly calculating the DFT and (b) using the FFT algorithm we discussed in class.

8. (10 points) In the final lab, you implemented a 6^{th} order IIR filter with 3 notches. Many students ran into stability issues, especially if they used r values above about 0.993-0.996. Explain why the stability problems were much more significant on the FM4 real-time hardware than in MATLAB. (Hint: It's related to the precision of each platform, but you must be more specific.)

7) Counts trivial mults.

(a) $N^2 = (2^{11})^2 = 2^{22} \sim 4$ million

(b) $N \log_2 N = 2048 \cdot 11 =$ 22,528

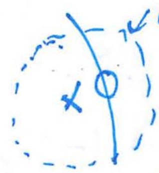
$$\begin{array}{r} 2048 \\ 2048 \\ \hline 22{,}528 \end{array}$$

Ratio $= \frac{2048}{11} \sim 185\times$

8) Lower precision → more coefficient movement → more root movement.

The zero-pole notch approach is very sensitive to root position.

drift of pole. unstable if $|p| \geq 1$



(-4) info on bit counts but not above detail

21

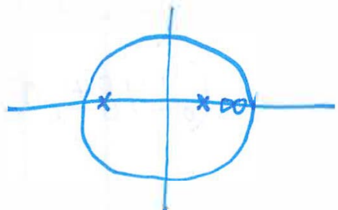
Given the difference equation y[n] = -0.4y[n-1] + 0.32y[n-2] + x[n] - 1.5 x[n-1] + 0.56 x[n-2]

9. (6 points) Take the z-transform of both sides of the equation and ***solve for the transfer function*** $H(z)$. Show your work. ~~(Note that x(n-1) does not appear in the expression.)~~
10. (10 points) ***Draw the pole-zero diagram*** for this system ***and*** (3)
    (4) a. Explain why the system is ***stable***.
    (3) b. ***Comment*** specifically on how these roots will affect the magnitude response.
11. (5 points) ***What form*** will the impulse response take? Hint: The impulse response can be found by taking the inverse z-transform of H(z), which benefits from partial fractions. Finding the ***form*** means you do not need to determine the parameters, but just the general expression. (e.g., A·sin(0.1πn)u(n) vs. 3.224 sin(0.1πn)u(n)).

9 $Y(z)\left(1+0.4z^{-1}-0.32z^{-2}\right) = X(z)\left(1-1.5z^{-1}+0.56z^{-2}\right)$

$$H(z)=\frac{Y(z)}{X(z)}=\frac{1-1.5z^{-1}+0.56z^{-2}}{1+0.4z^{-1}-0.32z^{-2}}=\frac{z^2-1.5z+0.56}{z^2+0.4z-0.32}$$

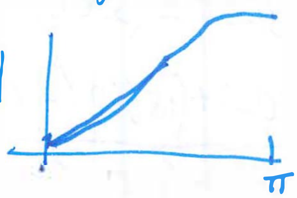
10 $H(z)=\dfrac{(z-0.8)(z-0.7)}{(z+0.8)(z-0.4)}$



a $p_i=\{-0.8, 0.4\}$, $\forall i, |p_i|<1$ (all response elements decay)

b 2 nearby 0s decrease LF gain, pole @ -0.8 increases HF gain.

Very roughly |H|



11 From 10a, & p_i

$h[n] = \left(A(-0.8)^n + B\cdot 0.4^n\right)u(n)$

-1 force $h[0]=0$ w/ $u(n-1)$

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Continuing the problem from the previous page...

12. (5 points) Determine the frequency response, H($e^{j\Omega}$), of this system.
13. (10 points) Given that the sampling frequency is 16 kHz and assuming ideal sampling, find the steady state digital output, y(n), given that the analog input is x(t) = cos(5600·2π·t - 60°).

(12) $H(e^{j\Omega}) = H(z)\Big|_{z=e^{j\Omega}} = \dfrac{e^{j2\Omega} - 1.5e^{j\Omega} + 0.56}{e^{j2\Omega} + 0.4e^{j\Omega} - 0.32}$

(13) $\Omega = \dfrac{F}{f_s} \cdot 2\pi = \dfrac{5600}{16000} \cdot 2\pi = \dfrac{56\pi}{80} = \dfrac{7\pi}{10}$ (-2 if wrong)

$H @ \dfrac{7\pi}{10} = \cdots \dfrac{(-0.309 - j0.951) + (0.882 - j1.214) + 0.56}{(-0.309 - j0.951) + (-0.235 + 0.324j) - 0.32} = \dfrac{1.133 - j2.165}{-0.864 - j0.627}$

$= 0.332 + j(2.265) = 2.289 \angle 1.425^R$

(7.2dB) (81.7°)

$x[n] = \cos\left(\frac{7\pi}{10} n - \frac{\pi}{3}\right)$

$y[n] = 2.289 \cos\left(\frac{7\pi}{10} n - 60° + 81.7°\right)$

$= 2.289 \cos\left(\underbrace{\frac{7\pi}{10}}_{2.199} n + \underbrace{21.7°}_{0.378^R}\right)$

(-1)