Milwaukee School of Engineering

Electrical Engineering and Computer Science Department

EE-3221 – Final Exam – Dr. Durant

Monday 22 February 2021

May use textbook (PDF or printed), 8½" × 11" note sheet, and calculator.

Good luck!

Name: Answers

Page 2:	(20 points)	______
Page 3:	(30 points)	______
Page 4:	(15 points)	______
Page 5:	(21 points)	______
Page 6:	(14 points)	______
Total:	(100 points)	______

20

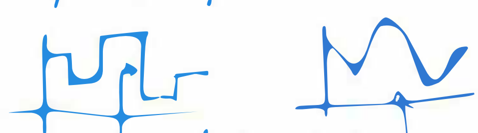
1. (5 points) Describe the purpose of the reconstruction filter in a DSP system in terms of the frequency content input and output.

2. (10 points) An analog signal has been lowpass filtered and contains frequencies between 0 and 4 kHz, but no components above 4 kHz remain. Without applying an anti-alias filter, the signal is sampled at rate of 6 kHz, resulting in aliasing. Explain what range of frequencies is corrupted by the aliasing ***and*** include sketches of the spectra of both the original analog signal and of the aliased signal.

3. (5 points) What is the period of $x(n) = \cos(\pi n/3) + \sin(2.1\pi n)$? Show your work.

① Remove energy $\geq f_s/2$ = Nyquist. This smooths the result + makes it consistent w/ the lowpass assumption

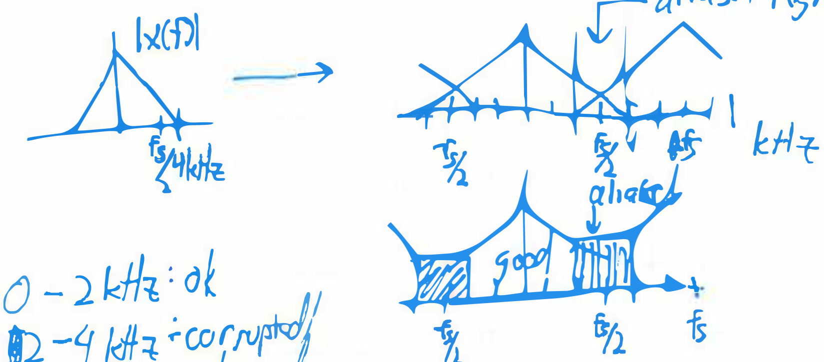
(-1) Vague on freq but generally correct
(-2) Lowpass not covered



- D/A intermediate output
- discontinuous
- all higher harmonics

No higher harmonics
No energy $> f_s/2$

②



(-2) >3 alias but not >2 corrupt

0 – 2 kHz : ok
2 – 4 kHz : corrupted/aliased

③ $\frac{\pi}{3} = \frac{k_1 2\pi}{N_1}$

$6k_1 = N_1$

$N_1 = 6,\ k_1 = 1$

(lowest integer sol.)

$2.1\pi = \frac{k_2 2\pi}{N_2}$

$2.1 N_2 = 2k_2$

$21 N_2 = 20 k_2$

$N_2 = 20\ \ k_2 = 21$

lowest int. sol.)

(-2) Mishandle 2.1π / inexact π
(-1) 6 & 20 but not LCM
(-2) $\frac{20}{21}$ rational vs. not

$N_0 = \mathrm{lcm}(N_1, N_2) = \mathrm{LCM}(6, 20) = \boxed{60}$

(30)

$x(n)$ $h(n)$

4. (10 points) Calculate the convolution [5 4 -1] * [3 7] directly using the convolution formula. Show your work.

5. (15 points) Recalculate the convolution using the FFT/IFFT method. Use the FFT that we learned in class (radix-2 decimation-in-time FFT). (-6) Missing 2 major steps or similar
(-3) Forgetting 1 major step

6. (5 points) Discuss whether your convolution results are consistent with the ***area property***.

(4)

15	12	-3	
	35	28	-7
15	47	25	-7

(5) $W_y = W_x + W_h - 1 = 3 + 2 - 1 = 4 \therefore N = 4$ for FFT

n	x(n)		X(k)
0	5	4	8
2	-1	6	$6-j4$
1	4	4	0
3		4	$6+j4$

Twiddle factors: 1, $-j$, -1, $+j$

n	h(n)		H(k)
0	3	3	10
2		3	$3-j7$
1	7	7	-4
3		7	$3+j7$

Twiddle factors: 1, $-j$, -1, $+j$

$W_4 = e^{-j\frac{2\pi}{4}} = -j$

$Y(k) = X(k) \cdot H(k) = [80 \quad -10-j54 \quad 0 \quad -10+j54]$

Just adjust for scaling by $N=4$. and use W_4 to negative powers (conjugate).

n	Y(n)			/N		
0	80	80	60	/4	=	15
2	0	80	188	/4	=	47
1	$-10-j54$	-20	100	/4	=	25
3	$-10+j54$	$-j108$	-28	/4	=	-7

Twiddle factors: 1, $+j$, -1, $-j$

} Matches 4

(6) $A_y = A_x \cdot A_h$

$\Sigma y_i = (\Sigma x_i) \cdot (\Sigma h_i)$

$15 + 47 + 25 - 7 = (5 + 4 - 1)(3 + 7)$

$80 = 10 \cdot 8$ ✓ ← yes. consistent.

(15)

7. (5 points) To create a notch filter we placed poles at $re^{\pm j\Omega_n}$, where r was approximately 0.995. What do these pole locations tell you about the form of the ***system impulse response***? (Hint: Table 7-5: z-Transform Pairs) (Hint: You do not need to consider the zeros to answer this question.)

8. (10 points) A filter is needed that has sharp transitions in the frequency response including 3 narrow notches. For this application, linear phase is ***not*** important. The filter is running on a very low power processor, so it is critical that the number of arithmetic operations be minimized. Would you choose an IIR or an FIR design? Defend whatever your choice is by explaining whether it helps meet each of the given design considerations.

(7) oscillating sinusoid w/ very slow decay.

Amplitude $= r^n = 0.995^n$

Frequency $= \Omega_n$ rad/sample

(-2) only states that it converges

(-2) freq. resp., not imp. resp.

(8) IIR

- Poles allow sharp increases independent of zero locations
- Poles let notches be arbitrarily narrow due to near-cancellation as in week 9 lab
- Poles cause nonlinear ϕ (except trivial poles @ $z = 0$)
- IIR tends to have lower order for a given freq. resp. shape.

(-2) if not mentioned but otherwise complete

21

Given the difference equation y(n) = -0.1 y(n-1) + 0.72 y(n-2) + 3 x(n) - 5 x(n-1) + 3 x(n-2)

9. (7 points) Take the z-transform of both sides of the equation and ***solve for the transfer function*** $H(z)$. Show your work. (-3) reciprocal
10. (7 points) ***Draw the pole-zero diagram*** for this system ***and comment*** specifically on how these roots are expected to affect the magnitude response.
11. (7 points) ***What form*** will the impulse response take? Hint: The impulse response can be found by taking the inverse z-transform of H(z), which benefits from partial fractions. Finding the ***form*** means you do not need to determine the parameters, but just the general expression. (e.g., A·sin(0.1πn)u(n) vs. 3.224 sin(0.1πn)u(n)).

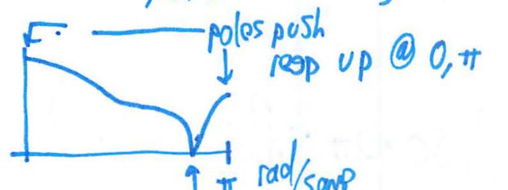
(9) $Y(z)(1+0.1z^{-1}-0.72z^{-2}) = X(z)(3-5z^{-1}+3z^{-2})$

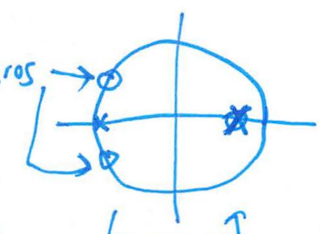
$$H(z) = \frac{Y(z)}{X(z)} = \frac{3-5z^{-1}+3z^{-2}}{1+0.1z^{-1}-0.72z^{-2}} \cdot \frac{z^2}{z^2} = \frac{3z^2-5z+3}{z^2+0.1z-0.72}$$

(10) Use calculator / quadratic formula to get roots.

Numerator roots = zeros = $+0.8333 \pm j0.5528 = 1\angle\pm 0.\cancel{8136}\,1864\pi$

Denominator " = poles = $-0.9, +0.8$





(11) $\frac{H(z)}{z}$ has no trivial zero to cancel so $\frac{A}{z} + \frac{B}{z+0.9} + \frac{C}{z-0.8}$

$$H(z) = A + \frac{Bz}{z+0.9} + \frac{Cz}{z-0.8}$$

$$h(n) = (A\delta(n) + \underbrace{B(-0.9)^n + C(0.8)^n}_{\text{sum of 2 geometric decays, 1 w/ alternating sign}})\,u(n)$$

~~DC offset~~
A=h(0)

(14)

Continuing the problem from the previous page…

12. (5 points) Determine the frequency response, $H(e^{j\Omega})$, of this system.
13. (9 points) Given that the sampling frequency is 8 kHz and assuming ideal sampling, find the steady state digital output, y(n), given that the analog input is x(t) = sin(750·2π·t - 30°).

(12) $z \to e^{j\Omega}$

$$H(e^{j\Omega}) = \frac{3e^{2j\Omega} - 5e^{j\Omega} + 3}{e^{2j\Omega} + 0.1e^{j\Omega} - 0.72}$$

(13)

3pts

$$\Omega = \frac{f}{f_s} 2\pi = \frac{750}{8000} \cdot 2\pi = \frac{3\pi}{16}$$

$$e^{j\Omega} = e^{j\frac{3\pi}{16}} \qquad e^{2j\Omega} = e^{j\frac{3\pi}{8}}$$

4pts

$$H(e^{j\Omega}) = \frac{3e^{j\frac{3\pi}{8}} - 5e^{j\frac{3\pi}{16}} + 3}{e^{j\frac{3\pi}{8}} + 0.1e^{j\frac{3\pi}{16}} - 0.72} = \frac{0.0112\angle -0.8125\pi}{1.0119\angle 0.5808\pi} = 0.0111\angle \overbrace{0.6067\pi}^{109^\circ}$$

2pts

$$y(n) = 0.0111 \sin\left(750 \cdot 2\pi \cdot \frac{n}{8000} + 79^\circ\right)$$

$$= 0.0111 \sin\left(\frac{3\pi n}{16} + 79^\circ\right)$$