Milwaukee School of Engineering

Electrical Engineering and Computer Science Department

EE-3220-31 – Final Exam – Dr. Durant

Wednesday 22 February 2017

May use 8½” × 11” note sheet and calculator for trig functions, etc.

Good luck!

Name: ____________________________________

Page 3:	(20 points)	______
Page 4:	(13 points)	______
Page 5:	(15 points)	______
Page 6:	(23 points)	______
Page 7:	(09 points)	______
Page 8:	(20 points)	______
Total:	(100 points)	______

Signal $x[n]$	z Transform $X(z)$	Region of Convergence
$\delta[n]$	1	all z
$u[n]$	$\frac{z}{z-1}$	$\lvert z\rvert > 1$
$\beta^n u[n]$	$\frac{z}{z-\beta}$	$\lvert z\rvert > \lvert\beta\rvert$
$nu[n]$	$\frac{z}{(z-1)^2}$	$\lvert z\rvert > 1$
$\cos(n\Omega)u[n]$	$\frac{z^2 - z\cos\Omega}{z^2 - 2z\cos\Omega + 1}$	$\lvert z\rvert > 1$
$\sin(n\Omega)u[n]$	$\frac{z\sin\Omega}{z^2 - 2z\cos\Omega + 1}$	$\lvert z\rvert > 1$
$\beta^n\cos(n\Omega)u[n]$	$\frac{z^2 - \beta z\cos\Omega}{z^2 - 2\beta z\cos\Omega + \beta^2}$	$\lvert z\rvert > \lvert\beta\rvert$
$\beta^n\sin(n\Omega)u[n]$	$\frac{\beta z\sin\Omega}{z^2 - 2\beta z\cos\Omega + \beta^2}$	$\lvert z\rvert > \lvert\beta\rvert$

- ω is the digital frequency, which is how many radians a sinusoid moves between samples.
- The DTFT is defined as $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n}$
- $X(e^{j(\omega+2\pi)}) = X(e^{j\omega})$
  - For real signals $X(e^{-j\omega}) = X^*(e^{j\omega})$.
  - The DTFT of a delayed signal, $y(n-k)$, is $e^{-j\omega k}Y(e^{j\omega})$.
- z^{-1} represents a sample delay
- The bilinear transform is $s = 2f_s\frac{z-1}{z+1}$
  - It results in frequency warping: $\Omega = 2f_s\tan\left(\frac{\omega}{2}\right)$, where Ω is in rad/s
- w_N gets a positive exponent in the forward DFT.

1. (5 points) Sketch the 5-component general DSP system diagram.

2. (3 points) What is the purpose of an anti-aliasing filter?

3. (6 points) A signal with continual, sinusoidal components at 3 kHz and 6 kHz is sampled at 10 kHz. Sketch the magnitude spectrum from -20 kHz to 20 kHz. The 3 kHz signal has greater energy.

4. (6 points) For each of the following 3 properties, give an example of a system equation (y(n) = f(x(n))) that satisfies it ***and exactly 1*** of the other 2 properties.

   a. Non-linear

   b. Time-invariant

   c. Causal

5. (4 points) Write the non-0 portion of the sequence resulting from $x(n) = \left(\frac{-1}{3}\right)^{n}(u(n+1)-u(n-3))$. Clearly indicate the *n=0* position in your sequence.

6. (4 points) Calculate the discrete-time Fourier transform (DTFT) of the x(n) you calculated above.

7. (5 points) Calculate the convolution [5 3 1] * [-2 4 7]. Show your work.

8. (5 points) Compare and contrast non-recursive and recursive filters. Why might one be desirable over the other and vice versa?

9. (10 points) Explain the relationship between the z-transform, the DTFT, and the DFT. Illustrate the relationship and difference using a sketch of the complex plane. Address the issues of transient and steady state response.

Given the difference equation y(n) = 0.75 y(n-1) – 0.5 x(n)

10. (5 points) Take the DTFT of both sides of the equation and solve for the transfer function $H\left(e^{j\omega}\right) = \frac{Y\left(e^{j\omega}\right)}{X\left(e^{j\omega}\right)}$

11. (3 points) Let f_s = 1000 Hz, f_1 = 150 Hz. Calculate the digital frequency, ω_1, for f_1.

12. (6 points) Evaluate $H\left(e^{j\omega}\right)$ at ω_1 ***and*** explain what it tells you about the system response.

13. (4 points) Determine the z-transform of the transfer function, H(z), of this system.

14. (5 point) Based on your H(z), determine an X(z) and the corresponding x(n) such that x(n) is a shortest possible non-zero signal that drives the system output y(n) to 0 by n = 1. Hint: The pole of H(z) causes the system to decay over time; you need to pick an input that cancels it.

15. (5 points) You want a digital filter with f_s = 30 kHz to have a corner frequency of 10 kHz, but want to design it using analog techniques and the bilinear transform. What should be the corner frequency of your prototype analog filter, to the nearest hertz? Show your work. Note whether your answer is greater or less than 10 kHz and whether this makes sense.

16. (4 points) List both an advantage and a disadvantage of a Chebyshev Type I versus a Butterworth lowpass filter design.

A 3.3 kHz analog sinusoid is sampled at 8 kHz. It is analyzed with a 64-point DFT.

17. (5 points) ***Determine*** where the ***peak(s)*** in the DFT magnitude spectrum will occur.

18. (6 points) What is the ***shortest length DFT that does not decrease the frequency resolution*** that will cause the signal to appear in a single DFT frequency bin?

19. (5 points) Sketch the *magnitude response* of the ideal lowpass sinc filter h(n) = sinc(n/6)/6.

20. (4 points) The sinc filter above is defined over all integers n. Describe the two steps commonly performed to allow it to be approximated in a real-time system.