Milwaukee School of Engineering

Electrical Engineering and Computer Science Department

EE-3220-31 – Final Exam – Dr. Durant

Wednesday 22 February 2017

For review, it will help your learning if you try the blank exam before reviewing these answers.

May use 8½" × 11" note sheet and calculator for trig functions, etc.

Good luck!

Name: Notes / Answers

Page 3:	(20 points)	______
Page 4:	(13 points)	______
Page 5:	(15 points)	______
Page 6:	(23 points)	______
Page 7:	(09 points)	______
Page 8:	(20 points)	______
Total:	(100 points)	______

Signal $x[n]$	z Transform $X(z)$	Region of Convergence
$\delta[n]$	1	all z
$u[n]$	$\dfrac{z}{z-1}$	$\lvert z\rvert > 1$
$\beta^n u[n]$	$\dfrac{z}{z-\beta}$	$\lvert z\rvert > \lvert\beta\rvert$
$nu[n]$	$\dfrac{z}{(z-1)^2}$	$\lvert z\rvert > 1$
$\cos(n\Omega)u[n]$	$\dfrac{z^2 - z\cos\Omega}{z^2 - 2z\cos\Omega + 1}$	$\lvert z\rvert > 1$
$\sin(n\Omega)u[n]$	$\dfrac{z\sin\Omega}{z^2 - 2z\cos\Omega + 1}$	$\lvert z\rvert > 1$
$\beta^n\cos(n\Omega)u[n]$	$\dfrac{z^2 - \beta z\cos\Omega}{z^2 - 2\beta z\cos\Omega + \beta^2}$	$\lvert z\rvert > \lvert\beta\rvert$
$\beta^n\sin(n\Omega)u[n]$	$\dfrac{\beta z\sin\Omega}{z^2 - 2\beta z\cos\Omega + \beta^2}$	$\lvert z\rvert > \lvert\beta\rvert$

- ω is the digital frequency, which is how many radians a sinusoid moves between samples.
- The DTFT is defined as $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n}$
- $X(e^{j(\omega+2\pi)}) = X(e^{j\omega})$
  - For real signals $X(e^{-j\omega}) = X^*(e^{j\omega})$.
  - The DTFT of a delayed signal, $y(n-k)$, is $e^{-j\omega k}Y(e^{j\omega})$.
- z^{-1} represents a sample delay
- The bilinear transform is $s = 2f_s\frac{z-1}{z+1}$
  - It results in frequency warping: $\Omega = 2f_s\tan\left(\frac{\omega}{2}\right)$, where Ω is in rad/s
- w_N gets a positive exponent in the forward DFT.

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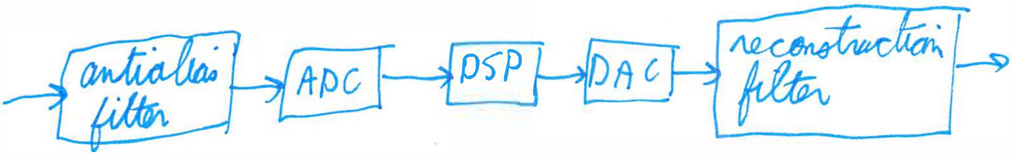
1. (5 points) Sketch the 5-component general DSP system diagram.

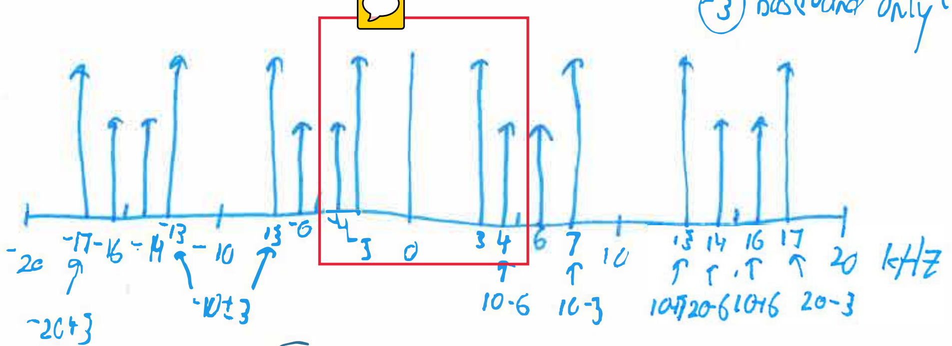


2. (3 points) What is the purpose of an anti-aliasing filter? (-2) "to prevent aliasing"

to prevent frequencies that would be confused for other frequencies due to sampled signals being periodic in frequency from entering the system. Typically, this is an LPF at $F_s/2$.

3. (6 points) A signal with continual, sinusoidal components at 3 kHz and 6 kHz is sampled at 10 kHz. Sketch the magnitude spectrum from -20 kHz to 20 kHz. The 3 kHz signal has greater energy.

(-2) missing conj.
(-3) baseband only (+1 w/ proper alias)



4. (6 points) For each of the following 3 properties, give an example of a system equation ($y(n) = f(x(n))$) that satisfies it and exactly 1 of the other 2 properties. 1pt 1pt

a. Non-linear

$y(n) = x^2(n-1) + n$

still causal

not-linear

time-varying

b. Time-invariant

$y(n) = \sum_{k=0}^{N-1} x(n-k)h(k)$

convolution is TI + linear. here, we give the causal form, which works for $h(n) = 0|_{n<0}$

c. Causal

$y(n) = nx^2(n-1)$

non-linear

causal

time varying

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on @ -1 off @ 3

5. (4 points) Write the non-0 portion of the sequence resulting from $x(n) = \left(\frac{-1}{3}\right)^n (u(n+1) - u(n-3))$. Clearly indicate the *n=0* position in your sequence.

$$x(n) = \left[\left(\frac{-1}{3}\right)^{-1} \left(\frac{-1}{3}\right)^{0} \left(\frac{-1}{3}\right)^{1} \left(\frac{-1}{3}\right)^{2} \right] = \left[-3 \ \underline{\underline{1}} \ \frac{-1}{3} \ \frac{1}{9} \right]$$

6. (4 points) Calculate the discrete-time Fourier transform (DTFT) of the x(n) you calculated above.

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n} = -3e^{j\omega} + 1 - \frac{1}{3}e^{-j\omega} + \frac{1}{9}e^{-2j\omega}$$

-2 extension of x(n) to ∞

7. (5 points) Calculate the convolution [5 3 1] * [-2 4 7]. Show your work.

x(n) h(n) ← e.g.

$$y(n) = \sum_{k=-\infty}^{\infty} x(k) h(n-k) \Rightarrow$$

n= 0	1	2	3	4	
5×-2	5×4	5×7			k = 0
	3×-2	3×4	3×7		k = 1
		1×-2	1×4	1×7	k = 2

-10	20	35		
	-6	12	21	
		-2	4	7

Σ = [$\underline{-10}$ 14 45 25 7]

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8. (5 points) Compare and contrast non-recursive and recursive filters. Why might one be desirable over the other and vice versa? Compare: both LTI implementations

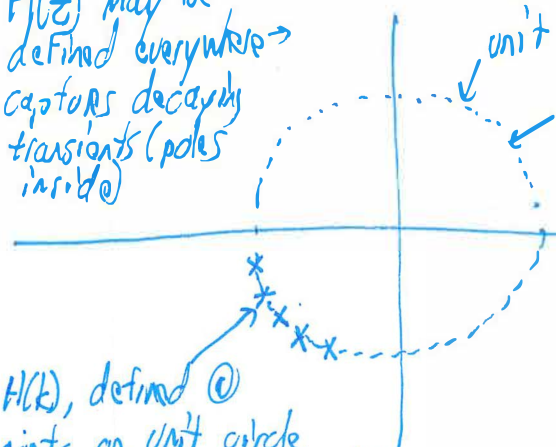
	pro	con
recursive	meet freq. spec @ lower order	poles can lead to instability using poles to drop mag. distorts linear phase
non-recursive	guaranteed stable easy to make linear phase (symmetric)	high order may be needed to match freq. spec → much memory + compute power

9. (10 points) Explain the relationship between the z-transform, the DTFT, and the DFT. Illustrate the relationship and difference using a sketch of the complex plane. Address the issues of transient and steady state response. OK to discuss in terms of signal and/or system

x, y (signal); h (system)

H(z) may be defined everywhere → captures decaying transients (poles inside)

unit circle



DTFT $H(e^{j\omega})$ defined on circle, free variable is ω, which is continuous (2π periodic due to sampling in time)

DFT, H(k), defined @ N points on unit circle, separated by resolution $\frac{2\pi}{N}$, $k = 0..(N-1)$. Since finite # of values, can be realized in hardware.

↑ signal length (N)

Domains

z:	$z \in \mathbb{C}$	(any complex #)	most general domain
DTFT:	$\omega \in \mathbb{R}$	(any real #)	↕
DFT	$k \in [0, N-1]$	(integers)	most constrained domain

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Given the difference equation y(n) = 0.75 y(n-1) – 0.5 x(n)

10. (5 points) Take the DTFT of both sides of the equation and solve for the transfer function $H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})}$

$Y(z) = \frac{3}{4} Y(z) z^{-1} - \frac{1}{2} X(z)$

$H(z) = \frac{Y(z)}{X(z)} = \frac{-1/2}{1 - 3/4 z^{-1}} \cdot \frac{z}{z} = \frac{-\frac{1}{2} z}{z - \frac{3}{4}}$

$H(e^{j\omega}) = H(z)\Big|_{z=e^{j\omega}} = \frac{-\frac{1}{2} e^{j\omega}}{e^{j\omega} - \frac{3}{4}} = \frac{-2e^{j\omega}}{4e^{j\omega} - 3} = \frac{-2}{4 - 3e^{-j\omega}}$ ← any similar form is okay

11. (3 points) Let f_s = 1000 Hz, f_1 = 150 Hz. Calculate the digital frequency, ω_1, for f_1.

$\omega_1 = \frac{f_1}{f_s} 2\pi = \frac{150}{1000} 2\pi = \frac{3}{20} 2\pi = \frac{3\pi}{10} \doteq 0.942 \frac{rad}{sec}$

↑ use this form

12. (6 points) Evaluate $H(e^{j\omega})$ at ω_1 ***and*** explain what it tells you about the system response.

$e^{-j\omega} = e^{-j\frac{3\pi}{10}} = 0.5878 - j0.8090$

$H(e^{j\omega}) = \frac{-2}{4 - 3(0.5878 - j0.8090)} = -0.4107 + j0.4456 = 0.6060\angle 2.3154$

$= 0.6060\angle 0.737\pi$

$= 0.6060\angle 132.7^\circ$

$= -4.35dB @ 132.7^\circ$

Response voltage is lowered to 0.606 of input + ~~advanced~~ delayed 132.7° (~~delayed~~ advancing 227.3°).

13. (4 points) Determine the z-transform of the transfer function, H(z), of this system.

see #10

14. (5 point) Based on your H(z), determine an X(z) and the corresponding x(n) such that x(n) is a shortest possible non-zero signal that drives the system output y(n) to 0 by n = 1. Hint: The pole of H(z) causes the system to decay over time; you need to pick an input that cancels it.

$X(z) = \underbrace{(z - \tfrac{3}{4})}_{\text{cancel pole}} \underbrace{z^{-1}}_{\text{make causal}} = 1 - \frac{3}{4} z^{-1}$

$x(n) = [\underline{1} \quad -\frac{3}{4}]$ ← or any constant multiple

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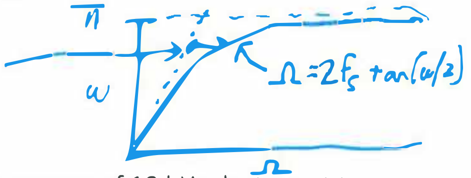
$\omega = \frac{f}{f_s} 2\pi = \frac{\Omega}{f_s}$



15. (5 points) You want a digital filter with f_s = 30 kHz to have a corner frequency of 10 kHz, but want to design it using analog techniques and the bilinear transform. What should be the corner frequency of your prototype analog filter, to the nearest hertz? Show your work. Note whether your answer is greater or less than 10 kHz and whether this makes sense.

$\omega = \frac{f}{f_s} 2\pi = \frac{10k}{30k} 2\pi = \frac{2\pi}{3}$

recip

$\Omega = 2f_s \tan(\omega/2) = 2 \cdot 30\text{kHz} \tan(\pi/3) = 60{,}000 \boxed{\frac{1/2}{\sqrt{3}/2}} = \frac{60{,}000\sqrt{3}}{\cancel{3}} \doteq 103923 \frac{\text{rad}}{s}$

$f = \Omega / 2\pi = \frac{30{,}000\sqrt{3}}{\pi} = \boxed{16{,}540\text{ Hz}}$ ← answer, > 10 kHz as expected, due to warping

Extra: $\Omega = 2f_s \tan(\omega/2) \to \omega = 2\tan^{-1}\left(\frac{\Omega}{2f_s}\right) = 2\tan^{-1}\left(\frac{10000 \cdot 2\pi}{2 \cdot 30000}\right) = 2\tan^{-1}\left(\frac{\pi}{3}\right) = 1.6169$

$\omega = 1.6169 = \frac{f}{f_s} 2\pi = \frac{f}{30000} 2\pi$

$f = 7{,}724$ Hz ← actual corner (digital) if you forget to prewarp

16. (4 points) List both an advantage and a disadvantage of a Chebyshev Type I versus a Butterworth lowpass filter design.

- faster transition to stop band : pro
- passband ripple : con

20

A 3.3 kHz analog sinusoid is sampled at 8 kHz. It is analyzed with a 64-point DFT.

17. (5 points) ***Determine*** where the ***peak(s)*** in the DFT magnitude spectrum will occur.

$$\frac{2\pi}{N}\cdot k = \frac{3.3k}{8k}\,2\pi$$

$$k = \frac{3.3}{8}N = \frac{3.3}{8}\cdot 64 = 8\cdot 3.3 = 26.4 \;\therefore$$ peak @ $\boxed{k=26}$

second peak at $N-k = 64-26 = \boxed{38}$ ← (-1) if missing

18. (6 points) What is the ***shortest length DFT that does not decrease the frequency resolution*** that will cause the signal to appear in a single DFT frequency bin?

$\frac{2\pi}{N}\cdot k = \frac{3.3kHz}{8kHz}2\pi$ has integer solution @ $N \geq 64$ due to resolution requirement.

$\frac{k}{N} = \frac{3.3}{8} = \frac{33}{80}$ any natural #

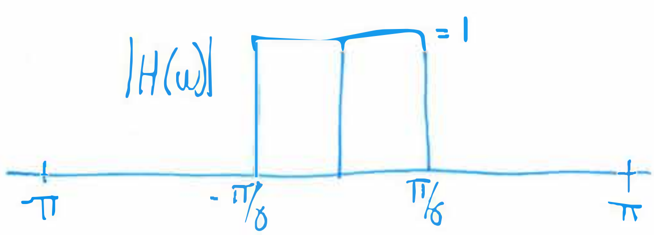
$N = \frac{80}{33}\cdot k \rightarrow N = 80m$

reduced

$\boxed{N=80}$ satisfies both constraints

19. (5 points) Sketch the *magnitude response* of the ideal lowpass sinc filter h(n) = sinc(n/6)/6.

$-\infty < n < \infty \;\therefore$ ideal LPF



20. (4 points) The sinc filter above is defined over all integers n. Describe the two steps commonly performed to allow it to be approximated in a real-time system.

1. truncate (symmetrically)
2. delay to make causal