Milwaukee School of Engineering

Electrical Engineering and Computer Science Department

# EE-3220-21 – Final Exam – Dr. Durant

Monday 24 February 2014

May use 3”×5” note card and nothing else.

***Good luck!***

Name: ____________________________________

| | | |
|---|---|---|
| Page 3: | (17 points) | ______ |
| Page 4: | (12 points) | ______ |
| Page 5: | (20 points) | ______ |
| Page 6: | (25 points) | ______ |
| Page 7: | (05 points) | ______ |
| Page 8: | (21 points) | ______ |
| Total: | (100 points) | ______ |

| Signal $x[n]$ | $z$ Transform $X(z)$ | Region of Convergence |
|---|---|---|
| $\delta[n]$ | $1$ | all $z$ |
| $u[n]$ | $\frac{z}{z-1}$ | $\|z\| > 1$ |
| $\beta^n u[n]$ | $\frac{z}{z-\beta}$ | $\|z\| > \|\beta\|$ |
| $nu[n]$ | $\frac{z}{(z-1)^2}$ | $\|z\| > 1$ |
| $\cos(n\Omega)u[n]$ | $\frac{z^2 - z\cos\Omega}{z^2 - 2z\cos\Omega + 1}$ | $\|z\| > 1$ |
| $\sin(n\Omega)u[n]$ | $\frac{z\sin\Omega}{z^2 - 2z\cos\Omega + 1}$ | $\|z\| > 1$ |
| $\beta^n\cos(n\Omega)u[n]$ | $\frac{z^2 - \beta z\cos\Omega}{z^2 - 2\beta z\cos\Omega + \beta^2}$ | $\|z\| > \|\beta\|$ |
| $\beta^n\sin(n\Omega)u[n]$ | $\frac{\beta z\sin\Omega}{z^2 - 2\beta z\cos\Omega + \beta^2}$ | $\|z\| > \|\beta\|$ |

- ω is the digital frequency, which is how many radians a sinusoid moves between samples.
- The DTFT is defined as $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n}$
- $X(e^{j(\omega+2\pi)}) = X(e^{j\omega})$
    - For real signals $X(e^{-j\omega}) = X^*(e^{j\omega})$.
    - The DTFT of a delayed signal, $y(n-k)$, is $e^{-j\omega k}Y(e^{j\omega})$.
- $z^{-1}$ represents a sample delay
- The bilinear transform is $s = 2f_s\frac{z-1}{z+1}$
    - It results in frequency warping: $\Omega = 2f_s\tan\left(\frac{\omega}{2}\right)$, where Ω is in rad/s
- $w_N$ gets a positive exponent in the forward DFT.

1. (3 points) Define “quantized” as it is applied to samples in a DSP system.

2. (3 points) What is the purpose of a reconstruction filter?

3. (6 points) A signal with continual, sinusoidal components at 4 kHz and 7 kHz is sampled at 10 kHz. Sketch the spectrum from -20 kHz to 20 kHz.

4. (5 points) Explain how, given knowledge of the analog input, you can tell from the spectrum of the sampled signal that aliasing occurred.

5. (6 points) For each of the following properties, give an example of a system equation (y(n) = f(x(n))) that satisfies it

   a. Non-linear

   b. Time-varying

   c. Non-causal

6. (3 points) Write the non-0 portion of the sequence resulting from $x(n) = (-2)^n(u(n+1)-u(n-2))$. Clearly indicate the *n=0* position in your sequence.

7. (3 points) Calculate the discrete-time Fourier transform (DTFT) of the x(n) you calculated above.

8. (5 points) Calculate the convolution [1 2 3] * [1 -2 5]. Show your work.

9. (5 points) Compare and contrast non-recursive and recursive filters. Why might one be desirable over the other and vice versa?

10. (10 points) Explain the relationship between the z-transform, the DTFT, and the DFT. Illustrate the relationship and difference using a sketch of the complex plane. Be sure to address the issues of transient and steady state response.

Given the difference equation y(n) = -0.6 y(n-1) + 0.4 x(n)

11. (5 points) Take the DTFT of both sides of the equation and solve for the transfer function $H\left(e^{j\omega}\right) = \frac{Y(e^{j\omega})}{X(e^{j\omega})}$

12. (2 points) Let $f_s$ = 1000 Hz, $f_1$ = 250 Hz. Calculate the digital frequency, $\omega_1$, for $f_1$.

13. (8 points) Evaluate $H\left(e^{j\omega}\right)$ at $\omega_1$ ***and*** explain what it tells you about the system response.

14. (5 points) Determine the z-transform of the transfer function, H(z), of this system.

15. (1 point) Let the input x(n) = [2 1.2] (non-zero at n=0 and n=1). Calculate X(z).

16. (4 points) Calculate Y(z) and simplify as much as possible.

17. (5 points) You want a digital filter with $f_s$ = 1 kHz to have a corner frequency of 250 Hz, but want to design it using analog techniques and the bilinear transform. What should be the corner frequency of your prototype analog filter, to the nearest hertz? Show your work. ($1/\pi$ = 0.318309886… will be helpful in your calculations.)

A 2 kHz analog sinusoid is sampled at 48 kHz. It is analyzed with a 64-point DFT.

18. (6 points) ***Determine*** where the ***peak(s)*** in the DFT magnitude spectrum will occur.

19. (5 points) What is the ***shortest length DFT that does not decrease the frequency resolution*** that will cause the signal to appear in a single DFT frequency bin?

20. (10 points) Calculate the DFT of [2 0 2 0], which equals [1 1 1 1]+[1 -1 1 -1].