

EE3220 DFT frequency resolution/density example, Dr. Durant 1/27/2015

The number of samples taken determines the frequency resolution. You cannot improve the resolution after acquiring your data.

Padding a signal with 0s and taking a longer DFT gives greater frequency density, which is sometimes desired since it gives a smoother graph, but it doesn't add information.

For example, 0.1 s of data are taken with f_s of 2 kHz. So, there are $N = 0.1 * 2000 = 200$ samples of data taken.

The resolution as a digital frequency $\omega = 2\pi/N$, which is equivalent to the frequency spacing for a DFT of length N . In this case $\omega = 2\pi/200 = 0.01\pi$ radians/sample. This can also be expressed in hertz: $(0.01\pi \text{ radians/sample}) * (2000 \text{ samples/s}) / (2\pi \text{ radians}) = 10 \text{ s}^{-1}$ or hertz. Note that N and f_s individually don't determine the resolution, but N/f_s , the duration measured, does. So, a shortcut is that the frequency resolution in hertz is the reciprocal of the duration in seconds.

Since the frequency resolution is exactly 10 Hz, we'll be able to see peaks in the spectrum for integer multiples of 10 Hz. Assume the input signal is the sum of sinusoids at 40 Hz & 70 Hz. Since these are multiples of 10 Hz, we'll see all their energy concentrated at $k = \{40, 70\} / 10 = \{4, 7\}$ in a 200-point DFT. The DFT is conjugate symmetric for real signals, so there are also peaks at $k = \{-4, -7\}$. However, DFT points are numbered from $k = 0$ to $k = N-1 = 199$, so we use frequency periodicity of the DFT and add $N = 200$ to the negative values to find that there will be peaks at $k = \{196, 193\}$.

Next, we 0-pad the 200-sample signal to a length of length 500 is and perform a DFT to increase density. The resolution is unchanged since we only collected 200 samples.

Since the DFT length is 2.5 times longer than the number of samples collected, the frequency density is 2.5 times "better" or smaller as well, $0.01\pi/2.5 = 0.004\pi$ radians/sample or 4 Hz.

Now, at what values of k do the 40 and 70 Hz components fall? Divide by the density of the DFT to find out. $k = \{40, 70\} / 4 = \{10, 17.5\}$. $k = 17.5$ doesn't work since the DFT is only defined for integer k . In this case, we don't have a sample exactly at 70 Hz, but samples at 68 and 72 Hz ($k = \{17, 18\}$). The frequency resolution is 10 Hz, which means that there is a DTFT peak at 70 Hz and 0s on either side at 60 & 80 Hz (& 50 & 90 Hz, etc.). 68 and 72 Hz are much closer to 70 Hz than to the 0s at 60 & 80 Hz, so we'll see large values at each of them. We take the negatives and add N to find the locations of all the peaks due to the conjugates. For example, for the 40 Hz component, there is also a peak at $N-k = 500-10 = 490$.

Demonstrating the above in MATLAB

```
T = 0.1; % s
fs = 2000; % Hz
N = T * fs; % samples taken
res_Hz = 1/T; % Hz

f = [40; 70]; % Hz
t = (0:N-1)/fs; % time of each sample
x = sum(cos(2*pi*f*t)); % f*t is an outer product - 1 row for each sinusoid, 1 column for each
time sample; sum sums the 2 sinusoids

figure,stem(0:N-1,abs(fft(x))),xlabel('Sample number k'),ylabel('|X|')
title('Example with density = resolution')

N2 = 500;
figure,stem(0:N2-1,abs(fft(x,N2))),xlabel('Sample number k'),ylabel('|X|')
title(sprintf('Example with density = %g \times resolution',N2/N))
```