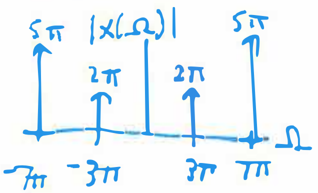
Name *Answers*

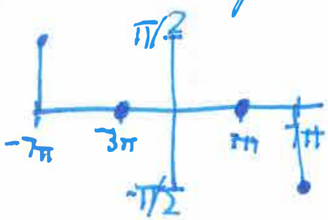
EE3032 – Dr. Durant – Quiz 9
Fall 2017, Week 9

1. (2 points) ***Find*** the Fourier transform, $X(\Omega)$, of $x(t) = 2\cos(3\pi t) + 5\sin(7\pi t)$.
2. (2 points) ***Sketch both*** the ***magnitude*** and the ***phase spectrum*** of $x(t)$.
3. (1 point) ***Sketch*** $|H(\Omega)|$ for an ideal lowpass filter with a cutoff frequency of 4π.
4. (2 points) Assume it is possible to implement this ideal filter in a real-time system and that your implementation has a delay of 7 s. (It is not possible to do this exactly, but you can get quite close.) What is $H(\Omega)$ for this filter? (Note: Your answer will be a piecewise definition and you will need to use the time shift property.)
5. (1 point) Find $Y(\Omega) = H(\Omega)\, X(\Omega)$.
6. (2 points) Find $y(t)$ by taking the inverse Fourier transform of $Y(\Omega)$.

(1) $X(\Omega) = 2\pi[\delta(\Omega-3\pi)+\delta(\Omega+3\pi)] - 5j\pi[\delta(\Omega-7\pi)-\delta(\Omega+7\pi)]$

(2)

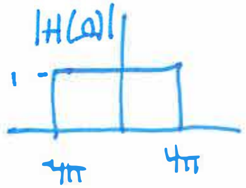




@ 7π, $\angle = \angle -j = -\pi/2$

@ -7π $\angle = -\angle -j = \pi/2$

(3)



(4) $H(\Omega) = \begin{cases} 1\angle -7\Omega = e^{-7j\Omega}, & |\Omega| < 4\pi \\ 0, & \text{otherwise} \end{cases}$

(5) $Y(\Omega) = H(\Omega)X(\Omega) = 2\pi e^{-7j\overset{3\pi}{\downarrow}}[\delta(\Omega-3\pi)+\delta(\Omega+3\pi)]$

(6) $y(t) = 2\cos(3\pi(t-7))$

Table 5.1

Basic Properties of Fourier Transform

	Time Domain	Frequency Domain
Signals and constants	$x(t), y(t), z(t), \alpha, \beta$	$X(\Omega), Y(\Omega), Z(\Omega)$
Linearity	$\alpha x(t) + \beta y(t)$	$\alpha X(\Omega) + \beta Y(\Omega)$
Expansion/contraction in time	$x(\alpha t), \alpha \neq 0$	$\frac{1}{\|\alpha\|} X\left(\frac{\Omega}{\alpha}\right)$
Reflection	$x(-t)$	$X(-\Omega)$
Parseval's energy relation	$E_x = \int_{-\infty}^{\infty} \|x(t)\|^2 dt$	$E_x = \frac{1}{2\pi} \int_{-\infty}^{\infty} \|X(\Omega)\|^2 d\Omega$
Duality	$X(t)$	$2\pi x(-\Omega)$
Time differentiation	$\frac{d^n x(t)}{dt^n}, n \geq 1$, integer	$(j\Omega)^n X(\Omega)$
Frequency differentiation	$-jtx(t)$	$\frac{dX(\Omega)}{d\Omega}$
Integration	$\int_{-\infty}^{t} x(t')dt'$	$\frac{X(\Omega)}{j\Omega} + \pi X(0)\delta(\Omega)$
Time shifting	$x(t - \alpha)$	$e^{-j\alpha\Omega} X(\Omega)$
Frequency shifting	$e^{j\Omega_0 t} x(t)$	$X(\Omega - \Omega_0)$
Modulation	$x(t)\cos(\Omega_c t)$	$0.5[X(\Omega - \Omega_c) + X(\Omega + \Omega_c)]$
Periodic signals	$x(t) = \sum_k X_k e^{jk\Omega_0 t}$	$X(\Omega) = \sum_k 2\pi X_k \delta(\Omega - k\Omega_0)$
Symmetry	$x(t)$ real	$\|X(\Omega)\| = \|X(-\Omega)\|$
		$\angle X(\Omega) = -\angle X(-\Omega)$
Convolution in time	$z(t) = [x * y](t)$	$Z(\Omega) = X(\Omega)Y(\Omega)$
Windowing/Multiplication	$x(t)y(t)$	$\frac{1}{2\pi}[X * Y](\Omega)$
Cosine transform	$x(t)$ even	$X(\Omega) = \int_{-\infty}^{\infty} x(t)\cos(\Omega t)dt$, real
Sine transform	$x(t)$ odd	$X(\Omega) = -j\int_{-\infty}^{\infty} x(t)\sin(\Omega t)dt$, imaginary

Table 5.2

Fourier Transform Pairs

	Function of Time	Function of Ω
(1)	$\delta(t)$	1
(2)	$\delta(t - \tau)$	$e^{-j\Omega\tau}$
(3)	$u(t)$	$\frac{1}{j\Omega} + \pi\delta(\Omega)$
(4)	$u(-t)$	$\frac{-1}{j\Omega} + \pi\delta(\Omega)$
(5)	$\text{sign}(t) = 2[u(t) - 0.5]$	$\frac{2}{j\Omega}$
(6)	$A, -\infty < t < \infty$	$2\pi A\delta(\Omega)$
(7)	$Ae^{-at}u(t), a > 0$	$\frac{A}{j\Omega + a}$
(8)	$Ate^{-at}u(t), a > 0$	$\frac{A}{(j\Omega + a)^2}$
(9)	$e^{-a\|t\|}, a > 0$	$\frac{2a}{a^2 + \Omega^2}$
(10)	$\cos(\Omega_0 t), -\infty < t < \infty$	$\pi[\delta(\Omega - \Omega_0) + \delta(\Omega + \Omega_0)]$
(11)	$\sin(\Omega_0 t), -\infty < t < \infty$	$-j\pi[\delta(\Omega - \Omega_0) - \delta(\Omega + \Omega_0)]$
(12)	$p(t) = A[u(t + \tau) - u(t - \tau)], \tau > 0$	$2A\tau \frac{\sin(\Omega\tau)}{\Omega\tau}$
(13)	$\frac{\sin(\Omega_0 t)}{\pi t}$	$P(\Omega) = u(\Omega + \Omega_0) - u(\Omega - \Omega_0)$
(14)	$x(t)\cos(\Omega_0 t)$	$0.5[X(\Omega - \Omega_0) + X(\Omega + \Omega_0)]$