Name Answers

EE3032 – Dr. Durant – Quiz 8
Fall 2019, Week 8

Convolution integral: $y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$ Transfer function: $H(\omega) = \int_{-\infty}^{\infty} h(t)e^{-j\omega t}dt$

Fourier series (periodic fcn.): $x(t) = \sum_{n=-\infty}^{\infty} X_n e^{jn\omega_0 t} = a_0 + 2\sum_{n=1}^{\infty} \alpha_n \cos(n\omega_0 t) - \beta_n \sin(n\omega_0 t)$

In the above, α and β are the real and imaginary parts of X (not a and b from the trigonometric form).

A real, periodic signal's Fourier Series terms include $X_0 = 5$, $X_1 = 3+j4$ and $X_2 = j7$. $X_n = 0$ for n>2.

1. (2 points) List all the other non-0 terms of X and their values.
2. (2 points) Calculate the power using Parseval's theorem, which states that the power is the sum of the squared magnitudes of ***all*** the Fourier Series coefficients. Show your work.
3. (2 points) Does the signal have even symmetry, odd symmetry, or neither? Explain your answer.
4. (2 points) Calculate the exponential term in the Fourier series associated with X_1. (Leave it in exponential form; in general, this will have a complex value.)
5. (1 point) Calculate a_1 and b_1 by converting X_1 accordingly. Recall that the trigonometric coefficients include the amplitude contributions from the positive ***and*** negative frequencies and are the coefficients of the appropriate cosine and sine terms.
6. (1 point) State the trigonometric terms in the signal due to a_1 and b_1 with correct amplitude, phase, and frequency.

① $X_{-1} = X_1^* = 3 - j4$ $X_{-2} = X_2^* = -j7$

② $P = \Sigma|X_n|^2 = |X_{-2}|^2 + |X_{-1}|^2 + |X_0|^2 + |X_1|^2 + |X_2|^2$

$= 7^2 + \underbrace{\left(\sqrt{3^2+4^2}\right)^2}_{=5} + 5^2 + 5^2 + 7^2 = 49+25+25+25+49 = 173$

③ Neither. Even symmetry ⟷ X_n is Real
Odd symmetry ⟷ X_n is Imaginary
But, X_n is complex here.

④ $X_1 e^{j1\omega_0 t} = (3+j4)e^{j\omega_0 t} = \left(5\angle \operatorname{atan}\left(\frac{4}{3}\right)\right)e^{j\omega_0 t} = 5e^{j\left(\omega_0 t + \operatorname{atan}\left(\frac{4}{3}\right)\right)}$
Any form is fine

⑤ $a_1 = 2\operatorname{Re}(X_1) = 6$ $b_1 = -2\operatorname{Im}(X_1) = -8$

⑥ $a_1 \cos(\omega_0 t) + b_1 \sin(\omega_0 t) = 6\cos(\omega_0 t) - 8\sin(\omega_0 t)$ $\{n=1\}$