Name *Answers*

EE3032 – Dr. Durant – Quiz 8
Fall 2017, Week 8

1. (3 points) Recall that the inverse Fourier transform is given by x(t) = 1/(2π) ∫ X(Ω) e^(jΩt) dΩ. State the formula for the (forward) Fourier transform. Hint: there are 2 (key) differences between the forward and inverse Fourier transforms.
2. (4 points) x(t) has Fourier transform X(Ω). y(t) = 2x(t-3) + 5. State Y(Ω) in terms of X(Ω).
3. (3 points) In your own words, explain what the time-frequency duality of the Fourier transform allows you to do. (You do not need details such as where the 2π factor goes. Note also that *duality* is different than the time-frequency *inverse* relationship. Partial credit will be given if you describe the *inverse* relationship instead of the *duality* relationship.

(1) $X(\Omega) = \int_{-\infty}^{\infty} x(t) e^{-j\Omega t}\, dt$

(2) Apply linearity twice (including scaling) + time delay

$$Y(\Omega) = \underbrace{2}_{\substack{\text{scale}\\ 2\times}}\underbrace{e^{-3j\Omega}}_{\substack{\text{delay}\\ 3s}} X(\Omega) + \underbrace{\overset{10}{\cancel{2}}\pi\delta(\Omega)}_{\substack{5\times \mathcal{F}\{1\}\\ \text{DC const.}}}$$

(3) TF duality allows the interchange of the roles of x(t) + X(Ω) in a FT pair to become $2\pi X(t) \Leftrightarrow x(-\Omega)$. (You do not need to show 2π + −Ω on the quiz.)

cap. small.

So, we can solve an FT pair if we recognize either x(t) or X(Ω) as a known T- or F-domain function.

e.g.,

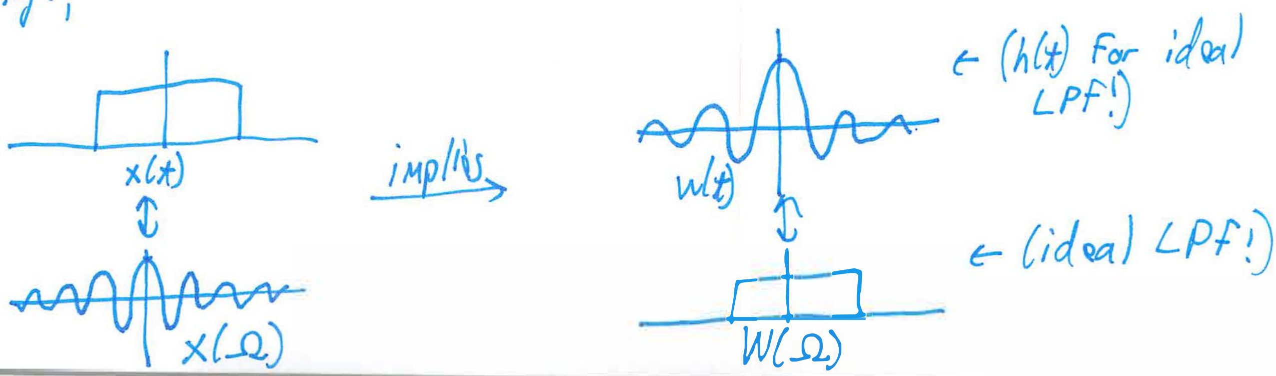

Table 5.1

Basic Properties of Fourier Transform

	Time Domain	Frequency Domain
Signals and constants	$x(t), y(t), z(t), \alpha, \beta$	$X(\Omega), Y(\Omega), Z(\Omega)$
Linearity	$\alpha x(t) + \beta y(t)$	$\alpha X(\Omega) + \beta Y(\Omega)$
Expansion/contraction in time	$x(\alpha t), \alpha \neq 0$	$\frac{1}{\lvert\alpha\rvert} X\left(\frac{\Omega}{\alpha}\right)$
Reflection	$x(-t)$	$X(-\Omega)$
Parseval's energy relation	$E_x = \int_{-\infty}^{\infty} \lvert x(t)\rvert^2 dt$	$E_x = \frac{1}{2\pi} \int_{-\infty}^{\infty} \lvert X(\Omega)\rvert^2 d\Omega$
Duality	$X(t)$	$2\pi x(-\Omega)$
Time differentiation	$\frac{d^n x(t)}{dt^n}, n \geq 1$, integer	$(j\Omega)^n X(\Omega)$
Frequency differentiation	$-jtx(t)$	$\frac{dX(\Omega)}{d\Omega}$
Integration	$\int_{-\infty}^{t} x(t')dt'$	$\frac{X(\Omega)}{j\Omega} + \pi X(0)\delta(\Omega)$
Time shifting	$x(t-\alpha)$	$e^{-j\alpha\Omega} X(\Omega)$
Frequency shifting	$e^{j\Omega_0 t} x(t)$	$X(\Omega - \Omega_0)$
Modulation	$x(t)\cos(\Omega_c t)$	$0.5[X(\Omega - \Omega_c) + X(\Omega + \Omega_c)]$
Periodic signals	$x(t) = \sum_k X_k e^{jk\Omega_0 t}$	$X(\Omega) = \sum_k 2\pi X_k \delta(\Omega - k\Omega_0)$
Symmetry	$x(t)$ real	$\lvert X(\Omega)\rvert = \lvert X(-\Omega)\rvert$
		$\angle X(\Omega) = -\angle X(-\Omega)$
Convolution in time	$z(t) = [x * y](t)$	$Z(\Omega) = X(\Omega)Y(\Omega)$
Windowing/Multiplication	$x(t)y(t)$	$\frac{1}{2\pi}[X * Y](\Omega)$
Cosine transform	$x(t)$ even	$X(\Omega) = \int_{-\infty}^{\infty} x(t)\cos(\Omega t)dt$, real
Sine transform	$x(t)$ odd	$X(\Omega) = -j\int_{-\infty}^{\infty} x(t)\sin(\Omega t)dt$, imaginary

Table 5.2

Fourier Transform Pairs

	Function of Time	Function of Ω
(1)	$\delta(t)$	1
(2)	$\delta(t-\tau)$	$e^{-j\Omega\tau}$
(3)	$u(t)$	$\frac{1}{j\Omega} + \pi\delta(\Omega)$
(4)	$u(-t)$	$\frac{-1}{j\Omega} + \pi\delta(\Omega)$
(5)	$\mathrm{sign}(t) = 2[u(t) - 0.5]$	$\frac{2}{j\Omega}$
(6)	$A, -\infty < t < \infty$	$2\pi A\delta(\Omega)$
(7)	$Ae^{-at}u(t), a > 0$	$\frac{A}{j\Omega + a}$
(8)	$Ate^{-at}u(t), a > 0$	$\frac{A}{(j\Omega + a)^2}$
(9)	$e^{-a\lvert t\rvert}, a > 0$	$\frac{2a}{a^2 + \Omega^2}$
(10)	$\cos(\Omega_0 t), -\infty < t < \infty$	$\pi[\delta(\Omega - \Omega_0) + \delta(\Omega + \Omega_0)]$
(11)	$\sin(\Omega_0 t), -\infty < t < \infty$	$-j\pi[\delta(\Omega - \Omega_0) - \delta(\Omega + \Omega_0)]$
(12)	$p(t) = A[u(t+\tau) - u(t-\tau)], \tau > 0$	$2A\tau \frac{\sin(\Omega\tau)}{\Omega\tau}$
(13)	$\frac{\sin(\Omega_0 t)}{\pi t}$	$P(\Omega) = u(\Omega + \Omega_0) - u(\Omega - \Omega_0)$
(14)	$x(t)\cos(\Omega_0 t)$	$0.5[X(\Omega - \Omega_0) + X(\Omega + \Omega_0)]$