Name Answers

EE3032 – Dr. Durant – Quiz 7
Winter 2019-2020, Week 7

Convolution integral: $y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$ Transfer function: $H(\omega) = \int_{-\infty}^{\infty} h(t)e^{-j\omega t}dt$

Fourier series (periodic fcn.): $x(t) = \sum_{n=-\infty}^{\infty} X_n e^{jn\omega_0 t} = a_0 + 2\sum_{n=1}^{\infty} \alpha_n \cos(n\omega_0 t) - \beta_n \sin(n\omega_0 t)$

$X_n = \alpha_n + j\beta_n$. Recall that in the phasor domain, where we derived the transfer function behavior by considering $x(t) = e^{+j\omega t}$, taking the derivative introduces a $j\omega$ term, where ω is a continuous variable.

1. (2 points) A system is described by $4\ddot{y}(t) - 3\dot{y}(t) + 2y(t) = 7\ddot{x}(t) + 5x(t)$. Convert to the phasor domain, using phasors Y and X.
2. (2 points) Simplify and collect like terms, rearranging the equation as $Y = H(\omega) \times X$ and solve for the transfer function $H(\omega)$.
3. (2 points) What is the system output if $x(t) = 7$?
4. 3 (~~2~~ points) What is the system output if $x(t) = 8 + 3\cos(4t)$?
5. 1 (~~2~~ points) Find all ω such that the output is in phase with the input.

① $4(j\omega)^2 Y - 3j\omega Y + 2Y = 7(j\omega)^2 X + 5X$

$Y(-4\omega^2 - 3j\omega + 2) = X(-7\omega^2 + 5)$

② $H = Y/X = \dfrac{-7\omega^2 + 5}{(2-4\omega^2) - 3j\omega}$

③ $H(0) = \dfrac{5}{2}$ $y(t) = H(0)\cdot x(t) = \frac{5}{2}\cdot 7 = \frac{35}{2} = 17\frac{1}{2}$ (DC output)

④ $H(4) = \dfrac{-7\cdot 16 + 5}{(2-4\cdot 16) - 3j4} = \dfrac{-107}{-62 - j12} = 1.6944\angle -0.1912$ OR $-10.95°$

$y(t) = \frac{5}{2}\cdot 8 + 3\cdot 1.6944\cos(4t - 10.95°)$

$= 20 + 5.0831\cos(4t - 10.95°)$

⑤ H must be real ∴ denominator of H must be real ∴ $-3\omega = 0$

$\omega = 0$ only.