Name Answers

EE3032 – Dr. Durant – Quiz 7
Fall 2019, Week 7

Convolution integral: $y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$ Transfer function: $H(\omega) = \int_{-\infty}^{\infty} h(t)e^{-j\omega t}dt$

Fourier series (periodic fcn.): $x(t) = \sum_{n=-\infty}^{\infty} X_n e^{jn\omega_0 t} = a_0 + 2\sum_{n=1}^{\infty} \alpha_n \cos(n\omega_0 t) - \beta_n \sin(n\omega_0 t)$

Recall that in the phasor domain, a derivative (of $e^{+j\omega t}$) introduces a jω term, where ω is a continuous variable. This can be seen in the Fourier series above, where frequency is discrete since x(t) is periodic.

1. (2 points) A system is described by $3\ddot{y}(t) + 2\dot{y}(t) - 4y(t) = 6x(t)$. Convert to the phasor domain, using phasors Y and X.
2. (2 points) Simplify and collect like terms, rearranging the equation as Y = H(ω) × X and solve for the (phasor) transfer function H(ω).
3. (2 points) What is the system output if x(t) = 3 cos(2t)?
4. (2 points) What is the system output if x(t) = 3 cos(2t) + 4 – 6 cos(3t)?
5. (2 points) Find all ω such that the output is in phase with the input.

① $3(j\omega)^2 Y + 2j\omega Y + -4Y = 6X$

② $(-3\omega^2 + 2j\omega - 4)Y = 6X$

$H = Y/X = 6/(2j\omega - (4+3\omega^2))$

③ $\omega = 2.\ H(2) = \dfrac{6}{j4-16} = \dfrac{3}{j2-8} = 0.3638\angle -2.8966$ $(-165.96°)$

$y(t) = 3 \cdot 0.3638(\cos(2t - 2.8966)) = 1.0914\cos(2t - 165.96°)$

④ 3 Different ω's. Use Superposition

$\omega = 0.\ H(0) = \dfrac{6}{-4} = -1.5$ (DC gain). $Y = HX = -1.5 \cdot 4 = -6$

$\omega = 3.\ H(3) = \dfrac{6}{j6-31} = -0.1866 - j0.0361 = 0.19\angle -2.9504$ $(-169.05°)$

$Y = HX = (0.19\angle -2.9504)(-6\angle 0) = -1.14\angle -169.05°$
$= 1.14\angle 10.95°$

$y(t) = 1.0914\cos(2t - 165.96°) - 6 + 1.14\cos(3t + 10.95°)$

⑤ $H(\omega)$ is real. $2\omega = 0 \therefore \boxed{\omega = 0}$