Name *Answer*

EE3032 – Dr. Durant – Quiz 6
Fall 2017, Week 6

1. (3 points) Find the ***fundamental frequency*** of x(t) = 4 + sin(2πt) – 3 cos(6πt).
2. (3 points) Find the continuous-time Fourier ***series,*** X_k, for x(t) from the previous problem.
3. (2 points) Show how you would ***modify*** X_k to get the CTFS of $x(-t)$, where x(t) is given above.
4. (2 points) What is the ***relationship*** between X_k ***and*** X_{-k} for any ***real***, periodic signal? (You do ***not*** need to prove this fact, but it can be proven starting with A cos($\Omega_0$kt) + B sin($\Omega_0$kt), which represents a sinusoid of the given frequency with any amplitude and phase shift, and applying Euler's formula.)

① $\Omega_1 = 0 \ \frac{rad}{s}$ $\quad \Omega_2 = 2\pi \quad \Omega_3 = 6\pi$

$F_1 = 0\ Hz \quad F_2 = 1 \quad F_3 = 3$

$T_1 = \infty\ s \quad T_2 = 1 \quad T_3 = 1/3$

$k=0$ regardless of Ω_0

$lcm(T_2, T_3) = 1 = T_0$

$\therefore \boxed{\Omega_0 = 2\pi}$

② $x(t) = 4 + \sin(\Omega_0 t) - 3\cos(3\Omega_0 t)$

$4 \rightarrow X_0$

$\sin(\Omega_0 t) \rightarrow \frac{-j}{2}(e^{j\Omega_0 t} - e^{-j\Omega_0 t})$

$-3\cos(3\Omega_0 t) \rightarrow \frac{-3}{2}(e^{j3\Omega_0 t} + e^{-j3\Omega_0 t})$

$X_1 = -j/2 \quad X_{-1} = j/2 \quad X_3 = \frac{-3}{2} \quad X_{-3} = \frac{-3}{2}$

$X = \{ \frac{-3}{2}, 0, \frac{j}{2}, \underline{4}, -j/2, 0, \frac{-3}{2} \}$

③ $X'_k = X_{-k}$ as proven in class, so fold:

$X' = \{ \frac{-3}{2}, 0, -j/2, \underline{4}, j/2, 0, \frac{-3}{2} \}$

④ $X_{-k} = X_k^*$.

Proof: $x_k(t) = A\cos(\Omega_0 kt) + B\sin(\Omega_0 kt) = \frac{A}{2}(e^{j\Omega_0 kt} + e^{-j\Omega_0 kt}) - \frac{jB}{2}(e^{j\Omega_0 kt} - e^{-j\Omega_0 kt})$

$= \underbrace{\left(\frac{A}{2} - j\frac{B}{2}\right)}_{X_k} e^{j\Omega_0 kt} + \underbrace{\left(\frac{A}{2} + j\frac{B}{2}\right)}_{X_{-k}} e^{-j\Omega_0 kt}$

$X_{-k} = X_k^*$ for any $x_k(t) = C\cos(\Omega_0 kt + \phi)$, Q.E.D.