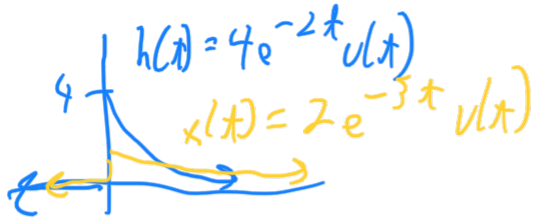
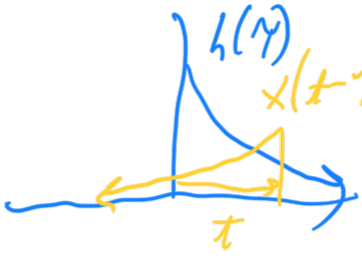
EE3032 WS Quiz Idea





Note: quiz should ask to fold h, not x, so graph will be slightly different.

$y(t) = h(t) * x(t)$

$h(t)$ is causal since $h(t) = 0 \; \forall \; t < 0$

$$= \int_0^t h(\tau) x(t-\tau) d\tau$$

$$= \int_0^t 4e^{-2\tau} 2e^{-3(t-\tau)} d\tau$$

$$= 8e^{-3t} \int_0^t e^{-(2+3)\tau} d\tau$$

$$= 8e^{-3t} \int_0^t e^{\tau} d\tau$$

$$= 8e^{-3t}(e^t - 1)$$

$$= 8(e^{-2t} - e^{-3t})$$

Max? $\dot{y}(t) = 8(-2e^{-2t} + 3e^{-3t}) = 0$

$2e^{-2t} = 3e^{-3t}$

$e^{t} = \frac{3}{2}$

$t^* = \ln\left(\frac{3}{2}\right)$

$y(t^*) = 8 \cdot \left(\frac{3}{2}\right)^{-3}\left(\frac{3}{2} - 1\right) = 4 \cdot \left(\frac{2}{3}\right)^3 = \frac{32}{27} = 1\frac{5}{27}$

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% EE3032 Winter 2019-20 Quiz 5

% Set up plotting/calculation time
dt = 0.01;
t = -1:dt:10;

% Express functions symbolically
syms tt
hh = 4 * exp(-2*tt) * heaviside(tt); % heaviside is MATLAB's name for the unit step,
u(t)
xx = 2 * exp(-3*tt) * heaviside(tt);
yy = 8 * exp(-3*tt) .* (exp(tt)-1) .* heaviside(tt);
% yy is y(t) found analytically. There is no *direct* way to get MATLAB to do
% the convolution symbolically, but there are some complex ways (Fourier
% transform properties) that we do not use here.

% Convert symbolic functions to functions for numeric evaluation, and evaluate
fh = matlabFunction(hh); h = fh(t);
fx = matlabFunction(xx); x = fx(t);
fy = matlabFunction(yy); y = fy(t);

% Perform the convolution numerically
yd = dt * conv(h,x);
tc = (2*t(1)) : dt : (2*t(end)); % domain expands per width property
yd = yd(tc>=t(1) & tc<=t(end)); % constrain to original domain

% Time and value of maximum of y(t) found analytically
tStar = log(3/2); % natural log, use log10 for base-10 log
yStar = fy(tStar);

% Plot and label results
figure
plot(t,h, t,x, t,y, t,yd)
hold on
plot(tStar, yStar, 'b*')
legend('h(t)', 'x(t)', 'y(t), analytic', 'y(t), numeric', 'max(y(t)), analytic')
xlabel('Time (s)'), ylabel('Voltage (V)')
title('EE3032 Quiz 5')

err = norm(y-yd) / sqrt(length(t)); % norm is vector length = Euclidean norm = RSSE
(root of sum of squared errors).
% Division converts to more standard RMSE (root of mean squared error) of y(t).
fprintf('RMSE between y(t) by analytic methods and numeric methods (truncated at %g s,
dt = %g) is %g.\n',...
    t(end), dt, err)
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