Name *Answers*

EE3032 – Dr. Durant – Quiz 5
Fall 2019, Week 5

Recall that the convolution integral is $y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$.

1. (1 point) Sketch x(t) = 2u(t) – 2u(t-3) and h(t) = e^{-4t} u(t)
2. (1 point) Explain whether the system is causal.
3. (1 point) Explain whether the system is linear and time-invariant.
4. (1 point) In the version of the integral above, note that h is flipped since the variable of integration appears with a negative sign. Recall that convolution is commutative, so the roles of x and h can be interchanged. Although you can ultimately get the correct answer either way, which function, x or h, does it make more sense to flip in this case? Why?
5. (2 points) As t goes to ∞, do you expect y(t) to approach 0, or some other constant? Explain your reasoning.
6. (4 points) Derive a closed form expression for y(t). Hint: there will be 2 regions with distinct expressions, not counting the region(s) where y(t) = 0.

① 

② Causal since $h(t)=0 \ \forall\ t<0$

③ Yes, since $h(t)$ exists (given)

④ Flip x since it is simpler. Mult by const, integrate over 3 seconds.

⑤ 0. The delayed $x(t-\tau)$ overlaps the tail of $h(\tau)$.

⑥ Region 1. $0\le t\le 3$. Shifted x has some $t<0$ values that are multiplied by 0 from h.
Region 2. $t\ge 3$. All 3 s of x overlap an active portion of h.

1: $y(t) = \int_0^t 2h(\tau)\,d\tau = \int_0^t 2e^{-4\tau}d\tau = -\frac{1}{4}2e^{-4\tau}\Big|_0^t = -\frac{1}{4}2(e^{-4t}-1) = \frac{1-e^{-4t}}{2}, \quad 0\le t\le 3$

2: $y(t) = \int_{t-3}^t 2h(\tau)d\tau = \int_{t-3}^t 2e^{-4\tau}d\tau = -\frac{1}{4}2e^{-4\tau}\Big|_{t-3}^t = -\frac{1}{4}2(e^{-4t}-e^{-4(t-3)})$

$= \frac{-e^{-4t}\cdot 2}{4}(1-e^{+12}) = \frac{e^{12}-1}{2}e^{-4t}, \quad t\ge 3$

And $y(t)=0,\ t\le 0$

Check area property

$$A_x = 2 \cdot 3 = 6$$

$$A_h = \int_0^\infty e^{-4t}\,dt = -\frac{1}{4} e^{-4t}\Big|_0^\infty = -\frac{1}{4}(0-1) = \frac{1}{4}$$

$$A_y = A_x \cdot A_h = 6 \cdot \frac{1}{4} = \frac{3}{2} = 1.5$$

$$\int_0^\infty y(t)\,dt = 3 \cdot \lim_{t\to\infty}\left(\frac{1-e^{-4t}}{2}\right) = 3 \cdot \frac{1}{2} = \frac{3}{2} = 1.5 \checkmark$$