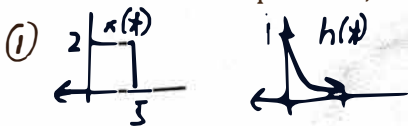


EE3032 - Dr. Durant - Quiz 5
Fall 2019, Week 5

Recall that the convolution integral is $y(t) = \int_{-\infty}^{\infty} x(\tau)h(t - \tau)d\tau$.

- (1 point) Sketch $x(t) = 2u(t) - 2u(t-3)$ and $h(t) = e^{-4t}u(t)$
- (1 point) Explain whether the system is causal.
- (1 point) Explain whether the system is linear and time-invariant.
- (1 point) In the version of the integral above, note that h is flipped since the variable of integration appears with a negative sign. Recall that convolution is commutative, so the roles of x and h can be interchanged. Although you can ultimately get the correct answer either way, which function, x or h , does it make more sense to flip in this case? Why?
- (2 points) As t goes to ∞ , do you expect $y(t)$ to approach 0, or some other constant? Explain your reasoning.
- (4 points) Derive a closed form expression for $y(t)$. Hint: there will be 2 regions with distinct expressions, not counting the region(s) where $y(t) = 0$.



② Causal since $h(t) = 0 \forall t < 0$

③ Yes, since $h(t)$ exists (given)

④ Flip x since it is simpler. Multiply constant ^{integrate} over 3 seconds.

⑤ 0. The delayed $x(t-\tau)$ overlaps the tail of $h(\tau)$.

⑥ Region 1. $0 \leq t \leq 3$. Shifted x has some $t < 0$ values that are multiplied by 0 from h .

Region 2. $t \geq 3$. All 3s of x overlap an active portion of h .

$$1: y(t) = \int_0^t 2h(\tau) d\tau = \int_0^t 2e^{-4\tau} d\tau = \left. -\frac{1}{2}e^{-4\tau} \right|_0^t = -\frac{1}{2}(e^{-4t} - 1) = \frac{1 - e^{-4t}}{2}, \quad 0 \leq t \leq 3$$

$$2: y(t) = \int_{t-3}^t 2h(\tau) d\tau = \int_{t-3}^t 2e^{-4\tau} d\tau = \left. -\frac{1}{2}e^{-4\tau} \right|_{t-3}^t = -\frac{1}{2}(e^{-4t} - e^{-4(t-3)})$$

$$= \frac{e^{-4(t-3)} - e^{-4t}}{2} = \frac{e^{12} - 1}{20} e^{-4t}, \quad t \geq 3$$

And $y(t) = 0, t \leq 0$

Check area property

$$A_x = 2 \cdot 3 = 6$$

$$A_h = \int_0^{\infty} e^{-4t} dt = \left. -\frac{1}{4} e^{-4t} \right|_0^{\infty} = -\frac{1}{4}(0-1) = \frac{1}{4}$$

$$A_y = A_x \cdot A_h = 6 \cdot \frac{1}{4} = \frac{3}{2} = 1.5$$

$$\int_0^{\infty} y(t) dt = 3 \cdot \lim_{t \rightarrow \infty} \left(\frac{1 - e^{-4t}}{2} \right) = 3 \cdot \frac{1}{2} = \frac{3}{2} = 1.5 \checkmark$$