Name Answers

EE3032 – Dr. Durant – Quiz 5
Winter 2019-2020, Week 5 <<Actually Quiz 4 / Week 4>>

Recall that the convolution integral is $y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$ and that convolution is commutative.

2 points

1. Circle those of the following system properties that are necessary for an impulse response to exist and, therefore, for convolution to apply: Linear, Time-invariant, ~~Causal~~, ~~BIBO Stable~~

3 points

2. Let h(t) be an impulse response that has non-zero values only between t=0 and 5 s. Let x(t) be a system input that has non-zero values only between t=-2 and 7 s.
   a. Using the width property, calculate when the output may have non-zero values.
   b. Explain why the system is or is not causal.

(a) h $[0, 5]$ x $[-2, 7]$

$0 + (-2)$, $5 + 7$

$[-2, 12]$

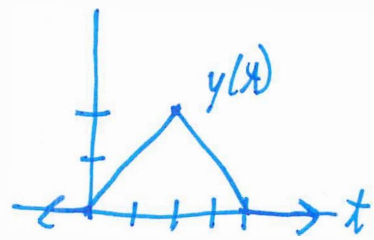
(b) Causal system $\Leftrightarrow$ $h(t) = 0 \ \forall t < 0$

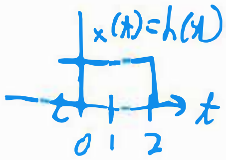
↑ "For all"

(turn over for problem 3)

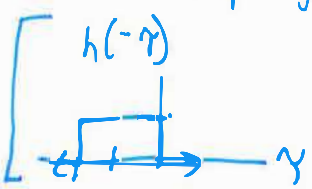
3. (5 points) Let x(t) = h(t) = u(t) – u(t-2). Calculate y(t) using convolution. You may use any combination of the graphical and analytical approaches. Show your work and fill in your final answer in the blanks below.

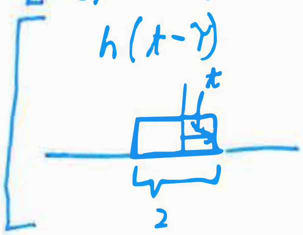
$$y(t) = \begin{cases} 0, & t \le 0 \\ t, & 0 \le t \le 2 \\ 4-t, & 2 \le t \le 4 \\ 0, & t \ge 4 \end{cases}$$





Width property: $[0,2]+[0,2]=[0,4] \therefore y(t)=0 \;\forall\; t\le 0 \vee t\ge 4$ ("OR")



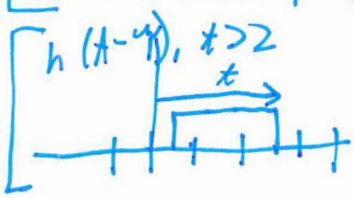


Partial, increasing overlap for $0\le t\le 2$

$y(t)=$ Area of overlap rectangle $=$ width $\times$ height

$=$ width $\times$ (height h $\times$ height x) $= t\cdot(1\cdot 1)=t$

Partial, decreasing overlap for $2\le t\le 4$



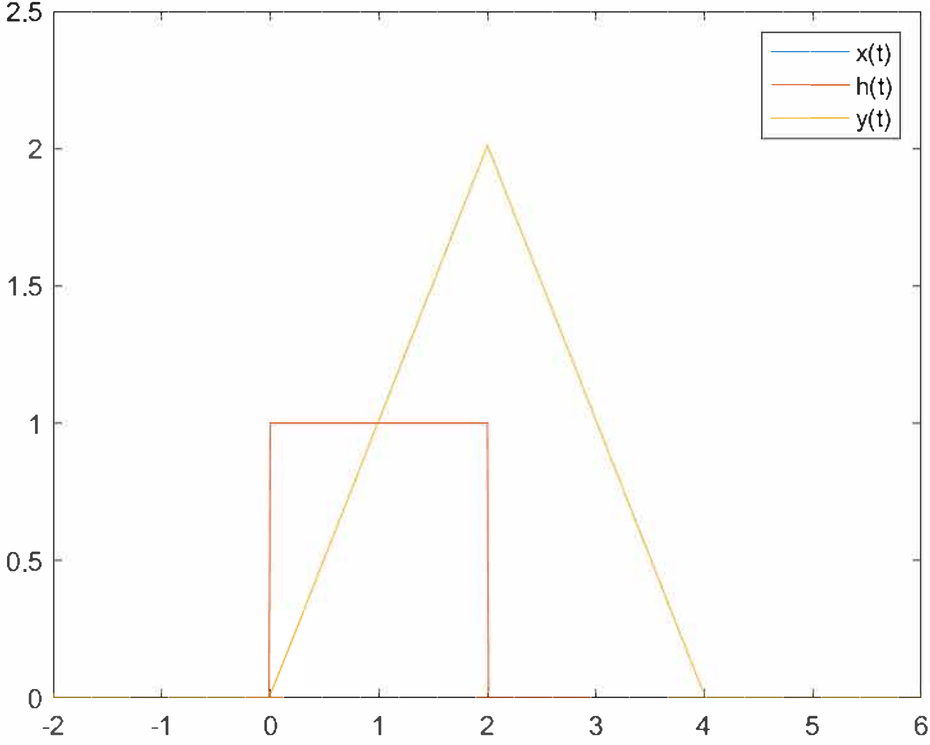
$y(t)=$ area of $(x(\tau)\cdot h(t-\tau))$

$=$ width $\times$ height $=$

$=\int_{t-2}^{2} 1\cdot 1\, d\tau = 2-(t-2) = 4-t$

```matlab
% EE3032 Winter 2019-20 Quiz 4 Problem 3 Solution

dt = 0.01;
t = -1:dt:3;
x = zeros(size(t));
x(t>=0 & t<=2) = 1;
h = x;
y = dt*conv(x,h);
ty = (2*t(1)):dt:(2*t(end)); % width property
plot(t,x, t,h, ty,y)
legend('x(t)', 'h(t)', 'y(t)')
```


x(t)
h(t)
y(t)
2.5
2
1.5
1
0.5
0
-2
-1
0
1
2
3
4
5
6